A method for CMOS IC design towards yield optimization

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Outline

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5. Remarks on CDF-based method implementation
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Introduction

- Importance of the yield optimization task
  - Time-to-market, and cost effectiveness,
  - Robustness of design;

- Corner (worst case – WC) methods
  - Fast design space exploration in terms of process and environment condition (PVT) variations,
  - Increase of number of corners; some worst-case corners may be missed,
  - Lack of correlations between different parameters sets;

- Monte-Carlo (MC) method
  - Time consuming, many simulations,
  - Does not actively manipulate / improve a design;

- Worst-case distance (WCD) method
  - Operates in process parameter space;

- Cumulative distribution function (CDF) – based method
  - Operates in design parameter space;
Given: length $L$ (design), sheet resistance $R_s$ (model) being a random variable of normal distribution:

$$R_s \sim N\left(R_{s,\text{mean}}, \sigma_{R_s}\right)$$

Task: Design a resistor, which fulfills the condition: $R_{\text{min}} < R < R_{\text{max}}$;

Find $W$ (design):

$$\max_W P\left(R_{\text{min}} \leq R \leq R_{\text{max}}\right) =$$

$$\max_W P\left(R_{\text{min}} \leq R_s \cdot \frac{L}{W} \leq R_{\text{max}}\right) =$$

$$\max_W P\left(R_{\text{min}} \cdot \frac{W}{L} \leq R_s \leq R_{\text{max}} \cdot \frac{W}{L}\right)$$
Conclusions:
1. yield depends on relation between parameter (R) constraints and process ($R_s$) quality;
2. cumulative distribution function (CDF) has been successfully used to determine optimum design.
Introductory example (2D case)

**Given:** sheet resistance $R_s$, resistor narrowing $\Delta W$ being uncorrelated random variables of normal distribution:

$$R_s \propto N \left( R_{s,\text{mean}}, \sigma_{R_s} \right) \quad \Delta W \propto N \left( \Delta W_{\text{mean}}, \sigma_{\Delta W} \right)$$

**Task:** Design a resistor, which fulfills the condition: $R_{\text{min}} < R < R_{\text{max}}$;

Find $W, L$ (design):

$$\text{max } P \left( R_{\text{min}} \leq R \leq R_{\text{max}} \right)$$

$$R_s = R_{s,\text{mean}} + \delta R_s$$

$$\Delta W = \Delta W_{\text{mean}} + \delta \Delta W$$

$$\delta R_s \propto N \left( 0, \sigma_{R_s} \right)$$

$$\delta \Delta W \propto N \left( 0, \sigma_{\Delta W} \right)$$

$$\text{max } P \left( R_{\text{min}} \leq R \leq R_{\text{max}} \right) = \text{max } P \left( R_{\text{min}} \leq R_s \cdot \frac{L}{W + \Delta W} \leq R_{\text{max}} \right) \approx$$

$$\text{max } P \left( \frac{R_{\text{min}} - \frac{R_{s,\text{mean}} \cdot L}{W + \Delta W_{\text{mean}}}}{\frac{R_{s,\text{mean}} \cdot L}{W + \Delta W_{\text{mean}}}} \leq \frac{\delta R_s - \frac{\delta \Delta W}{W + \Delta W_{\text{mean}}}}{\frac{R_{s,\text{mean}} \cdot L}{W + \Delta W_{\text{mean}}}} \leq \frac{R_{\text{max}} - \frac{R_{s,\text{mean}} \cdot L}{W + \Delta W_{\text{mean}}}}{\frac{R_{s,\text{mean}} \cdot L}{W + \Delta W_{\text{mean}}}} \right)$$
## Introductory example (2D)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
<th>Value 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{\text{min}}$ ($\Omega$)</td>
<td>980</td>
<td>940</td>
<td>940</td>
</tr>
<tr>
<td>$R_{\text{max}}$ ($\Omega$)</td>
<td>1020</td>
<td>1060</td>
<td>1060</td>
</tr>
<tr>
<td>$R_{s,\text{mean}}$ ($\Omega$/sq)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$\sigma_{R_s}$ ($\Omega$/sq)</td>
<td>0.3</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>$\Delta W_{\text{mean}}$ (m)</td>
<td>-2.0E-7</td>
<td>-2.0E-7</td>
<td>-2.0E-7</td>
</tr>
<tr>
<td>$\sigma_{\Delta W}$ (m)</td>
<td>3.0E-8</td>
<td>3.0E-8</td>
<td>1.0E-8</td>
</tr>
</tbody>
</table>

![Graphs showing length versus width for different values of length](image)
CDF-based approach

- **D** - design parameter vector
- **M** - process parameter vector
- **X** - circuit performance vector
- **S** – specification vector

**Yield optimization task**

\[
D_{\text{opt}} = \arg \max_D \ P(X \in S)
\]

\[
X = f(D, M)
\]

\[
X \in S \equiv \begin{cases} 
X_{1,\text{min}} \leq X_1(D, M) \leq X_{1,\text{max}} \\
X_{2,\text{min}} \leq X_2(D, M) \leq X_{2,\text{max}} \\
\vdots \\
X_{N_X,\text{min}} \leq X_{N_X}(D, M) \leq X_{N_X,\text{max}} 
\end{cases}
\]

Partial yields
CDF-based approach

\[ X_i(D,M) = X_i(D,M_{\text{nom}} + \delta M) \approx X_i(D,M_{\text{nom}}) + \sum_{j=1}^{NM} \left. \frac{\partial X_i}{\partial M_j} \right|_{(D,M_{\text{nom}})} \delta M_j \]

Nominal design i-th performance  
Sensitivities  
Process par. variations

Remarks:

Relation to BPV method used for statistical modelling, i.e. for extraction of \( \delta M \) – random var.

Issues:

To determine \( M_{\text{nom}} \)

To determine \( \delta M \)

Remarks:

D\text{opt} should maximize joint probability:

\[
P\left( \bigcap_{i=1}^{NX} \left( X_{i,\text{min}} - X_i(D,M_{\text{nom}}) \leq \sum_{j=1}^{NM} \left. \frac{\partial X_i}{\partial M_j} \right|_{(D,M_{\text{nom}})} \delta M_j \leq X_{i,\text{max}} - X_i(D,M_{\text{nom}}) \right) \right)
\]

An optimization problem has been formulated

Issues:

Select reliable method for yield optimization
CDF-based approach

I. If $M_j$ are uncorrelated normally-distributed random variables, then

$$Y = \sum_{j=1}^{NM} \frac{\partial X_i}{\partial M_j} \delta M_j \sim N(0, \sigma_Y), \quad \sigma_Y^2 = \sum_{j=1}^{NM} \left( \frac{\partial X_i}{\partial M_j} |_{(D,M_{\text{nom}})} \right)^2 \cdot \sigma_{M_j}^2$$

**Issues:**
- Select uncorrelated process parameters
- Process parameters not normally distributed

II. Due to the **unavoidable** correlation between performances, direct yield and product of partial yields are not equal.

$$P\left( \bigcap_{i=1}^{N_X} (X_{i,\text{min}} \leq X_i(D,M) \leq X_{i,\text{max}}) \right) \neq \prod_{i=1}^{N_X} P(X_{i,\text{min}} \leq X_i(D,M) \leq X_{i,\text{max}})$$

**Issue:** Account for correlations between performances, or assume, that

$$P\left( \bigcap_{i=1}^{N_X} (X_{i,\text{min}} \leq X_i(D,M) \leq X_{i,\text{max}}) \right) = \prod_{i=1}^{N_X} P(X_{i,\text{min}} \leq X_i(D,M) \leq X_{i,\text{max}})$$
CDF-based approach

Based on assumptions I, II a joint probability (parametric yield) may be calculated as a product of CDFs $F_i$ of normal distributions

$$\prod_{i=1}^{N_X} \int_{x_i\text{,min}}^{x_i\text{,max}} f_i(x_i) \, dx_i = \prod_{i=1}^{N_X} \left[ F_i \left( x_i\text{,max} - x_i(D,M_{\text{nom}}) \right) - F_i \left( x_i\text{,min} - x_i(D,M_{\text{nom}}) \right) \right]$$

**Interpretation**

If $N_X=1$ case is considered, the task may be illustrated "geometrically"

Maximize shaded area
Backward Propagation of Variance Method

Calculations of standard deviations of process parameters based on variations of performances and on performance sensitivities.


- Selection of non-correlated process parameters ($m$)
- Selection of PCM performances ($e$)

\[
\begin{pmatrix}
\sigma^2_{Vth(P)} \\
\sigma^2_{Idsat(P)} \\
\sigma^2_{gm(P)} \\
\sigma^2_{rds(P)} \\
\sigma^2_{Vth(N)} \\
\sigma^2_{Idsat(N)} \\
\sigma^2_{gm(N)} \\
\sigma^2_{rds(P)} \\
\end{pmatrix}
\begin{pmatrix}
S_{11} & S_{12} & \cdots \\
S_{21} & S_{22} & \cdots \\
\vdots & \vdots & \ddots \\
\end{pmatrix}
\begin{pmatrix}
\sigma^2_{TOX} \\
\sigma^2_{lnNSUB(P)} \\
\sigma^2_{UO(P)} \\
\sigma^2_{DL(P)} \\
\sigma^2_{DW(P)} \\
\sigma^2_{lnNSUB(N)} \\
\vdots \\
\end{pmatrix}
\]

\[
\sigma^2_e = S \sigma^2_m ; \quad S_{ij} = \left( \frac{\partial e_i}{\partial m_j} \right)^2
\]
CDF-based approach vs BPV method

- Functional block performance (PCM) sensitivities @ nominal process parameters
- Functional block performance variances determined experimentally

CDF of functional block performance variances

BPV

- Process parameter variances
- Functional block performance sensitivities @ nominal process parameters
- Process parameter variances
CDF method - Inverter

Inverter performances J.P.Uemura, "CMOS Logic Circuit Design", Kluwer, 2002

Inverter threshold

\[
V_I = \frac{V_{DD} - |V_{T,p}| + \sqrt{\frac{\beta_n}{\beta_p}} \cdot V_{T,n}}{1 + \sqrt{\frac{\beta_n}{\beta_p}}}
\]

\[I_{DD,max} = I_{DD}(V_{in} = V_I)\]

Propagation delay

\[
t_{P,HL} = \left[ \frac{2V_{T,n}}{V_{DD} - V_{T,n}} + \ln \left( 4 \frac{V_{DD} - V_{T,n}}{V_{DD}} - 1 \right) \right] \cdot R_n \cdot C_{out}
\]

\[
t_{P,LH} = \left[ \frac{2|V_{T,p}|}{V_{DD} - |V_{T,p}|} + \ln \left( 4 \frac{V_{DD} - |V_{T,p}|}{V_{DD}} - 1 \right) \right] \cdot R_p \cdot C_{out}
\]

\[
t_P = 0.5 \cdot (t_{P,HL} + t_{P,LH})
\]
CDF method – OpAmp

OpAmp performances

Low-frequency gain
\[ A_v = \left( \frac{g_{m2}}{g_{o2} + g_{o4}} \right) \cdot \left( \frac{g_{m6}}{g_{o6} + g_{o7}} \right) \]

Phase margin
\[ PM = \Pi - \sum_{j=1}^{4} \arctan \left( \frac{\omega_c}{p_j} \right) \]

Equivalent input-referred noise power spectral density
\[ S_{in}^2 = S_1^2 + S_2^2 + \left( \frac{g_{m3}}{g_{m1}} \right)^2 \cdot \left( S_3^2 + S_4^2 \right) \]

- \( g_{mi}, g_{oi} \) - input and output conductances of i-th transistor,
- \( \omega_c \) is a unity-gain bandwidth,
- \( p_j \) - j-th pole of the circuit,
- \( S_k \) - input-referred noise power spectral densities, consisting of thermal and 1/f components.
CDF method – Inverter, OpAmp

- Monte-Carlo method 1000 samples
- simple MOSFET model
- 0.8 µm CMOS technology
  - $t_{ox} = 20$ nm,
  - $V_{thn} = 0.7$ V, $V_{thp} = -0.9$ V
- process parameters varied:
  - gate oxide thickness $t_{ox}$,
  - substrate doping conc. $N_{subn}$, $N_{subp}$,
  - carrier mobilities $\mu_{on}$, $\mu_{op}$,
  - fixed charge densities $N_{ssn}$, $N_{ssp}$
CDF method - Inverter

Contour plots of yield in design parameter space

open - partial yields
closed - product of partial yields (CDF)
solid - product of partial yields (CDF)
dashed - product of partial yields (MC)
dotted - yield (MC)

A "valley" results from the specification of $t_{P,\text{min}}$ constrain.
CDF method – OpAmp

Contour plots of yield in design parameter space

open - partial yields
closed - product of partial yields (CDF)
solid - product of partial yields (CDF)
dashed - product of partial yields (MC)
dotted - yield (MC)
CDF-based method implementation

Objective function maximization

- Close to the maximum the objective function may exhibit a plateau;

Optimization task based on gradient approach requires in this case 2nd order derivatives of yield function, but...

this makes optimization based on gradient methods useless;
CDF-based method implementation

Objective function maximization

- Objective function may exhibit more than one plateau or more local maxima;

thus

- a non-gradient global optimization method is required.
Conclusions, further work (1)

1. The presented CDF-based method may be used for IC block design optimizing parametric yield,

2. The method may predict parametric yield of the design,

3. The design rules of the given IP and also discrete set of allowed solutions* may be directly used and shown in the yield plots in the design parameter space,

4. The method may be very useful for evaluation if the process is efficient enough to achieve a given yield,

5. The CDF-based method gives results, which are very close to the Monte Carlo method,

6. The results of yield optimization ($Y_{opt}$) based on CDF method have direct interpretation in design parameter space (problem of selection of design parameters: explicit or combined),

* Pierre Dautriche, "Analog Design Trends & Challenges in 28 and 20 nm CMOS Technology", ESSDERC'2011
Conclusions, further work (2)

7. If the performance constraints are mild with respect to process variability, a continuous set of design parameters, for which yield close to 100% is expected,

8. If the constraints are severe with respect to process variability, the method leads to unique solution, for which the parametric yield below 100% is expected,

9. The method may be used for performances determined both analytically, as well as via Spice-like simulations (batch mode) – to be done,

10. Method should be developed to take into account performance correlations and non-gaussian distributions – to be done,

11. The methodology may be used for design types (not only of ICs) taking into account statistical variability of a process and aimed at yield optimization.
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