Interference and Distortion Analysis for Nonlinear Analog Circuits
Overview

- Motivation
  - Electromagnetic Interferences, Susceptibility / Immunity

- Distortion Analysis
  - Nonlinear Effects, Intermodulations

- MOSFET Power Series Expansion
  - EKV current equation

- Block-Model for Interference Analysis
  - Multi-Input Wiener-Model

- Obtaining Polynomial I/O-Descriptions
  - Characterizing static nonlinearities, Example: Diff.-Amp.
Motivation (1)

Electromagnetic Compatibility (EMC)

Considerations about the mutual Interaction or influence of electrical systems caused by unwanted electric or electromagnetic effects

Susceptibility

Inability of a device or system to perform without Degradation in the presence of Electromagnetic disturbances

Immunity

Ability of a device or system to perform without degradation in the presence of electromagnetic disturbances

Motivation (2)
Design Specification for Integrated Analog Circuit

E.G. PSRR: Power Supply Rejection Ratio

\[ \text{PSRR}^+ = \frac{U_{\text{out}}(j\omega)}{U_{\text{in}}(j\omega)} \]
\[ \text{PSRR}^- = \frac{U_{\text{out}}(j\omega)}{U_{\text{in}}(j\omega)} \]

→ Intermodulations between Signal and Distortion

Simulation (AC / linear)

BUT:
Nonlinear Analysis

Signal: 100\(\mu\)V@10kHz, Distortion: 100\(\mu\)V @ 500Hz

PSRR \(~ -100\text{dB}\)
If interference signals couple into a CMOS analog circuit, the output signal will show spurious components arising from intermodulations between interference signal and nominal input signal.

Goal: derive analytical expressions for the amplitudes of distortion components.
Static Nonlinearities

- Nonlinear characteristics give rise to distortions at the output

- Distortions are spurious components in output spectrum
  - One-Tone excitation: Harmonic Distortion (at multiples of inp. frequencies)
  - Multi-Tone excitation: Intermodulation Distortion (mixing products of inp. freq.)
Power Series Approach: Single Tone, Figures of Merit

Procedure for Single Input (Power series approach):

- Obtain a polynomial approximation for the transfer characteristics

\[ y(t) = a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) \ldots \]

\( y(t) = Y_0 - \tilde{y}(t) \) and \( x(t) = X_0 - \tilde{x}(t) \)

\((X_0, Y_0)\) Operating Point

- Substitute excitation signal and calculate response for each order

\[ x(t) = \hat{X} \cos(\omega t); \quad \omega = 2\pi f \]

\[ \Rightarrow \quad y(t) = \left( \frac{a_2}{2} \hat{X}^2 \right) + \left( a_1 + \frac{3}{4} \hat{X}^2 \right) \hat{X} \cos(\omega t) + \frac{a_2}{2} \hat{X}^2 \cos(2\omega t) + \frac{a_3}{2} \hat{X}^3 \cos(3\omega t) + \ldots \]

- The coefficients of the n-th power of polynomial description corresponds to amplitude of the n-th order distortion component

Figures of merit: Harmonic Distortion

\[ HD_2 = \frac{a_2}{2a_1} \hat{X} \]

\[ HD_3 = \frac{a_3}{4a_1} \hat{X}^2 \]

(Defined as the ratio of n-th order component to fundamental tone at the output)
Power Series Approach: Multi-Tone

- Same procedure as for single-tone case, but a multi-frequency signal is substituted.
- E.g., for a two-tone signal with equal amplitudes:

\[ x(t) = \hat{X} \left( \cos(\omega_1 t) + \cos(\omega_2 t) \right) \]
\[ \Rightarrow y(t) = a_2 \hat{X}^2 + \left( a_1 + \frac{9a_3}{4} \hat{X}^2 \right) \hat{X} \left[ \cos(\omega_1 t) + \cos(\omega_2 t) \right] + \frac{a_2}{2} \hat{X}^2 \left[ \cos(2\omega_1 t) + \cos(2\omega_2 t) \right] \]
\[ + \frac{a_3}{4} \hat{X}^3 \left[ \cos(3\omega_1 t) + \cos(3\omega_2 t) \right] + a_2 \hat{X}^2 \left[ \cos(\omega_1 t - \omega_2 t) + \cos(\omega_1 t + \omega_2 t) \right] \]
\[ + \frac{3a_3}{4} \hat{X}^3 \left[ \cos(2\omega_1 t - \omega_2 t) + \cos(2\omega_1 t + \omega_2 t) + \cos(\omega_1 t - 2\omega_2 t) + \cos(\omega_1 t + 2\omega_2 t) \right] \]

- The coefficients of the n-th power of polynomial description corresponds to amplitude of the n-th order mixing product.

Figures of merit: IM-Distortion

\[ \text{IM}_2 = \frac{a_2}{a_1} \hat{X} = 2 \text{HD}_2 \]
\[ \text{IM}_3 = \frac{3a_3}{4a_1} \hat{X}^2 = 3 \text{HD}_3 \]

(Defined as the ratio of n-th order IM component to fundamental tone at the output)
Static Power Series: General Case

- Nonlinear transfer characteristics \( y(t) = f_{NL}(x(t)) = a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \ldots \)

- General signal with Q distinct frequencies \( x(t) = \sum_{q=1}^{Q} \hat{X}_q \cos(\omega_q t) \)

- Multiplication in time domain corresponds to convolution in freq. Domain

\[ Y(j\omega) = a_1 X(j\omega) + a_2 X(j\omega) * X(j\omega) + a_3 X(j\omega) * X(j\omega) * X(j\omega) + \ldots \]

  - 1st order response
  - 2nd order response
  - 3rd order response

- Each mixing frequency is a linear combination of excitation frequencies

\[ \omega_m = \sum_{q=-Q}^{Q} m_q \omega_q \quad \text{with} \quad \sum_q m_q = n \]
Obtaining Power Series for MOSFET Current

\[ i_{DS} = \mu C_{ox} \frac{W}{L} (u_{GS} - U_{th})^2 \]

Modelled by a VCCS:

Third order IM components occur close to the fundamental and are hence important measures in distortion analysis.

Third order components can not be accounted for by using a quadratic model (third and higher derivatives vanish!)

Quadratic model is only valid in active operating region (i.e. \( U_{GS} > U_{th} \), \( U_{DS} > U_{GS} - U_{th} \))

Using EKV-Model, which is valid for all \( U_{GS} > 0 \)

Taylor Series expansion

\[ I_{DS0} - i_{DS} = a_1 (U_{GS0} - u_{GS}) + a_2 (U_{GS0} - u_{GS})^2 \]

with \( a_m = \frac{1}{m!} \frac{\partial^m i_{DS}}{\partial u_{GS}^m} \bigg|_{u_{GS}=U_{GS0}} \)

\((U_{GS0}, I_{DS0})\) operating point
Using EKV Current Equation

The (modified) EKV current equation:

\[ i_{DS} = 2 \frac{W}{L} \mu C_{ox} U_t^2 \left[ \log^2 \left( 1 + e^{\left( \kappa (u_G - U_{th}) - u_S \right) / 2U_t} \right) - \log^2 \left( 1 + e^{\left( \kappa (u_G - U_{th}) - u_D \right) / 2U_t} \right) \right] \]

- Possibility to cope with higher order distortion components
- More tedious to calculate derivatives in Taylor expansion (symbolic calculations done with MAPLE)
- Complicated to compare with circuit simulations (e.g. SPECTRE), since only small signal parameters are available
  - Introduce fitting parameter and using threshold voltage in current equation
Accuracy - Comparison to Circuit Simulators

\[ i_{DS} = 2 \frac{W}{L} \mu C_{ox} U_t^2 \log^2 \left( 1 + e^{(\kappa (u_G - U_{th}) - u_S) / 2U_t} \right) \]

fitting parameter: \( \kappa = 0.92 \)

SPECTRE DC Analysis
EKV Equation

\[ U_{th} = 0.4979 \]
Taylor Series Expansion using EKV-Model

- Because of EKV-Characteristic, an arbitrary operating point can be used.
- The higher the order of Taylor series, the better the accuracy of approximation (of course, only in the vicinity of operating point).
- Using EKV, higher order derivatives > 2 can be taken into account.

![Graph showing Taylor series expansion using EKV-Model](image)

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Multi-Input Distortion Analysis

- Interference signal is regarded as distortion source at additional input
  - Need for Multi-Input Analysis
- Circuits with multiple nonlinearities
- Goal: find the output signal in terms of both inputs in form of a multivariate power series
Incorporating Dynamic Effects

- So far, only static nonlinearities has been considered

- If the system’s I/O behavior contains memory, modelling becomes more complex (cf. “Volterra series“)

- Easiest way to consider dynamic effects: system is dividable into a linear dynamic part and a nonlinear static part (cf. “Wiener-/Hammerstein-Model“)

E.G.: common source small signal model
General Volterra Approach for nonlinear Systems

\[ H_n[x(t)] = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} h_n(\tau_1, \ldots, \tau_n) x(t - \tau_1) \cdots x(t - \tau_n) d\tau_1 \cdots d\tau_n \]

**Input signal + Interference**

\[ x(t) \]

**Multidimensional Fourier-Transformation**

**n-th order Volterra-Kernel**

\[ H_n(j\omega_1, \ldots, j\omega_n) \]

**Output signal**

\[ y(t) = H_1[x(t)] + H_2[x(t)] + \ldots + H_n[x(t)] + \ldots \]

**Output Signall + Distortion Effects**

\[ y(t) \]
The blocks $H_{in}(f)$ and $H_{dis}(f)$ are purely linear blocks, completely described by linear transfer function.

The frequency band in which a given circuit is most susceptible to EMI is given by frequency range in which $H_{dis}$ exhibits its highest gain.

$F(x_{in}, x_{dis})$ constitutes the nonlinear behavior in form of a multivariate polynomial description.
Block-Model Approach (2)

Linear blocks:

- \( H_{in} \) for input linear block
- \( H_{dis} \) for disturbance linear block

Nonlinear block:
- Multivariate polynomial
  \( F(\tilde{x}_{in}, \tilde{x}_{dis}) \)

Polynomial nonlinearity:

\[
\tilde{y}(t) = \sum_{i=0}^{N_1} \sum_{j=0}^{N_2} a_{ij} \tilde{x}_{in}^i(t) \tilde{x}_{dis}^j(t) = a_0 + \sum_{i=0}^{N_1} a_i \tilde{x}_{in}^i(t) + \sum_{j=0}^{N_2} a_0j \tilde{x}_{dis}^j(t) + \sum_{k=1}^{N_1} \sum_{l=1}^{N_2} a_{kl} \tilde{x}_{in}^k(t) \tilde{x}_{dis}^l(t)
\]

Splitting up distortion effects:

- \( F_1 \) for signal
- \( F_2 \) for interference
- \( F_3 \) for interference

(Cf. Wiener/Hammerstein-Model)

Signal

Interference

IM: Signal/Interference
Characterization of the Blocks

- \( H_{in} \) and \( H_{dis} \) can be characterized by linear small signal analysis.

- Characterization of static block is more complex.
  - Procedure: Determine the multivariate polynomial by a Volterra approach.
  - Only static elements will be considered (nonlinear R/G, controlled sources).

Example: Differential Amplifier with distorted \( U_{DD} \)

\[
H_{dis}(f_{dis}) = \frac{U_{GS3}}{U_{dis}}
\]
Approach: Method of Nonlinear Sources (1)

Substitute nonlinear element by linearized element in parallel to nonlinear sources.

**Volterra-Series**

\[ i(t) = \sum_{n=1}^{N} c^n i_n(t) = \sum_{m=1}^{M} a_m u^m(t) = \sum_{m=1}^{M} a_m \left[ \sum_{n=1}^{N} c^n u_n(t) \right]^m \]

**Polynomial Approximation**

Sort coefficients of equal powers of c

\[ i_1(t) = a_1 u_1(t) \]
\[ i_2(t) = a_1 u_2(t) + a_2 u_1^2(t) \]
\[ i_3(t) = a_1 u_3(t) + 2a_2 u_1(t) u_2(t) + a_3 u_1^3(t) \]
\[ \vdots \]
\[ i_n(t) = i_n,linear(t) + i_n,nonlinear(u_1, \ldots, u_n) \]
Approach: Method of Nonlinear Sources (2)

- Substitute nonlinear elements
- Successively solve for every order $n$
  - 1st order: excitation sources
  - $n>2$: nonlinear current sources
- Sum up all $n$-th order responses to obtain approximation for I/O-behavior
  - Output function is a power series in terms of its input
  - Calculation of Volterra kernels not necessary
Example: Differential Amplifier (1)
Characterizing the Linear Blocks

- Small signal analysis of linearized circuit

Because of frequency characteristics of $H_{\text{dis}}$ und $H_{\text{sig}}$:

- Distortion components can be generated in signal frequency band
- For low order distortion, the components at $f_{\text{in}} \pm f_{\text{dis}}$ (2nd order) and $f_{\text{in}} \pm 2f_{\text{dis}}$ (3rd order) will occur close to fundamental at $f_{\text{in}}$
**Example: Differential Amplifier (2)**

**Characterizing the Nonlinear Block**

- Polynomial approximation of static nonlinear elements (Taylor series expansion)
- Determining 1st order response from excitation signals (input and interference signal)
- Determining n-th order response from nonlinear sources
- In the end, we obtain a polynomial I/O-description in which the coefficients will be given in terms of the n.-l. coefficients of the n.-l. elements
Analytical expression of overall polynomial static transfer function:

\[ u_{out}(u_{in}, u_{dis}) = K_{1,0} u_{in} + K_{3,0} u_{in}^3 + K_{1,1} u_{in} u_{dis} + K_{1,2} u_{in} u_{dis}^2 \]

Analytical Expressions:

\[ K_{1,0} = -2Rg_m \]
\[ K_{3,0} = 2R \alpha_{3,1} + \frac{8RR_ds a_{2,1}}{1 + 2R_ds a_{1,1}} \]
\[ K_{1,1} = \frac{4RR_ds a_{2,1} a_{1,2}}{1 + 2R_ds a_{1,1}} \]
\[ K_{1,2} = -\frac{4R_ds a_{2,1} a_{2,2}}{1 + 2R_ds a_{1,1}} + \frac{6R^2_ds a_{3,1} a_{1,2}^2}{(1 + 2R_ds a_{1,1})^2} + \frac{8R^3_ds a_{2,1}^2 a_{1,2}^2}{(1 + 2R_ds a_{1,1})^3} \]

(Here: \( a_{n,i} \) is the n-th order Taylor coefficient of i-th Transistor)
Diff.-Amp.: Intermodulations at the Output Compared to SPECTRE (QPSS Analysis)

Analytical Expressions:

- Amplitude at $f_{in} \pm f_{dis}$:
  \[ A_{2nd} = K_{1,1} |H_{dis} U_{dis}| |H_{in} U_{in}| \]

- Amplitude at $f_{in} \pm 2f_{dis}$:
  \[ A_{3rd} = K_{1,2} |H_{dis} U_{dis}|^2 |H_{in} U_{in}| \]

Figures of Merit: IM-Distortion

\[ \text{IM}_2 = \frac{K_{1,1}}{K_{1,0}} |U_{dis} H_{dis}(f_{in} \pm f_{dis})| \]

\[ \text{IM}_3 = \frac{K_{1,2}}{K_{1,0}} |U_{dis} H_{dis}(f_{in} \pm 2f_{dis})|^2 \]
Diff.-Amp.: Output Spectrum
Compared to SPECTRE (QPSS Analysis)

- $u_{\text{dis}} = 1\text{mV}@5.8\text{kHz}$
- $u_{\text{in}} = 10\text{mV}@40\text{kHz}$

- Good accuracy compared to SPECTRE
- Second harmonic not included in block model
- Deviations because of precision of nonlinearity coefficients
Summary and Outlook

- Interference and distortion effects in nonlinear analog circuits
  - Power Series Approach

- Block model for EMI analysis
  - Splitting up linear dynamic and static nonlinear part

- Method of nonlinear current for characterization of nonlinear part
  - Multivariate polynomial in terms of input and interference signal

- Analytical expressions for intermodulation distortions

Outlook / Conclusion

- Splitting linear dynamics and static nonlinearity not always possible from circuit structure
  - Using approximations (e.g. mean square sense, system identification techniques)

- Accuracy considerations of power series of device characteristics
  - Always a problem in power series approaches
  - Carry out the method using Chebyshev polynomials
The End
Optaining Power Series for Circuit-Blocks

- Interference signal is regarded as distortion source at additional input
- Goal: find the output \( u_{\text{out}}(t) \) in terms of \( u_{\text{in}}(t) \) and \( u_{\text{dis}}(t) \) in form of a multivariate power series
Even order distortion

E.g.: first order (quadratic) MOSFET Model

Polynomial approximation: $y = a_1 x + a_2 x^2$
Odd order distortion (symmetry)

E.g.: differential pair characteristics

Polynomial approximation: $y = a_1 x + a_3 x^3$
Beispiel:
Signal: Tiefpass
Störung: Bandpass

Block-Modell:
- Lineare Blöcke legen Frequenzbereich der max. Störanfälligkeit fest (max. Verstärkung von $H_{\text{dis}}$)
- Nichtlineare Effekte:
  Intermodulationen (Störung/Signal) können in das Signalband fallen → Nichtlinearer Anteil
Approach: Method of Nonlinear Currents

- According to Volterra Series (Summation of responses)

\[ i(t) = \sum_{n} i_n(t) \]
\[ u(t) = \sum_{n} u_n(t) \]

\[ u(t) = \sum_{n=1}^{N} c^n u_n(t) = \sum_{m=1}^{M} a_m i^m(t) = \sum_{m=1}^{M} a_m \left[ \sum_{n=1}^{N} c^n i_n(t) \right]^m \]

- Sort coefficients of equal powers of \( c \)

- Nonlinear Element is substituted by linearized element in parallel to nonlinear current sources
Example: Differential Amplifier (2)

Output signal:

\[ u_{out}(t) = R \cdot i_{out}(t) \]
\[ i_{out}(t) = i_{out,1} - i_{out,2} = \sum_{n=1}^{N} i_{out,n}(t) \]

Analytical expression of overall polynomial static transfer function:

\[ u_{out}(u_{in}, u_{dis}) = K_{1,0}u_{in} + K_{3,0}u_{in}^3 + K_{1,1}u_{in}u_{dis} + K_{1,2}u_{in}u_{dis}^2 \]