



Analytical modeling of nano MOSFETs in the quasi ballistic regime : beyond the drift diffusion approximation

R. Clerc,
G. Ghibaudo

P. Palestri,
L. Selmi

IMEP-LAHC
Grenoble Institute of Technology
France

DIEGM
University of Udine
Italy





Transport in Nano MOSFET ($L < 20$ nm) challenges Compact Modeling

Since the pioneering work of [Natori](#) (1994) and [Lundstrom](#) (1996), the quasi ballistic regime of transport in nano MOSFETs has been extensively investigated.

However, existing compact model are still based on Drift Diffusion and saturation velocity concepts

*« Sophisticated computer simulations using techniques such as full band Monte Carlo and full quantum transport approaches are being used to explore the physics of the ultimate MOSFET, but **circuit models continue to be based on concepts and approaches developed in the 1960's.** »*

M. Lundstrom, Int. SOI Conference, 2006

→ Let us examine the applicability of Quasi Ballistic theories to compact modeling



Outline

The "orthodox" Lundstrom theory

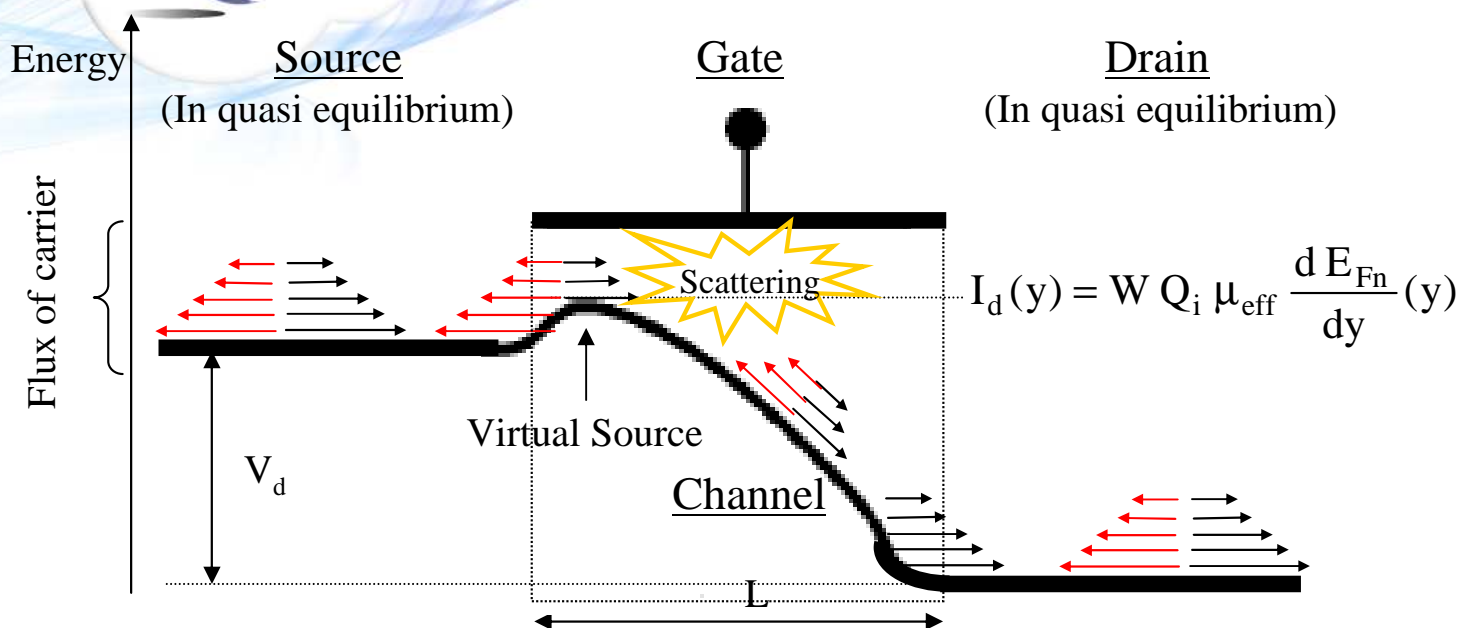
Beyond Lundstrom's theory

Ballisticity extraction from experiments

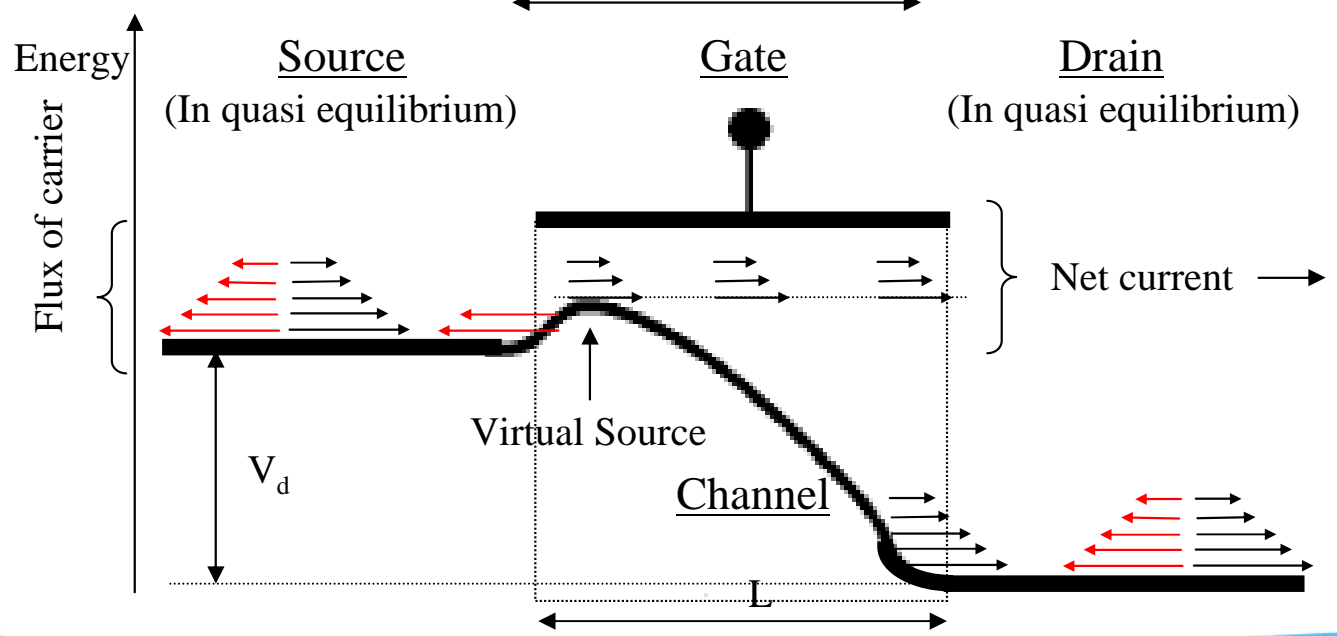


The "orthodox" Lundstrom theory

Concept of Ballistic Limit



Transistor in diffusive regime

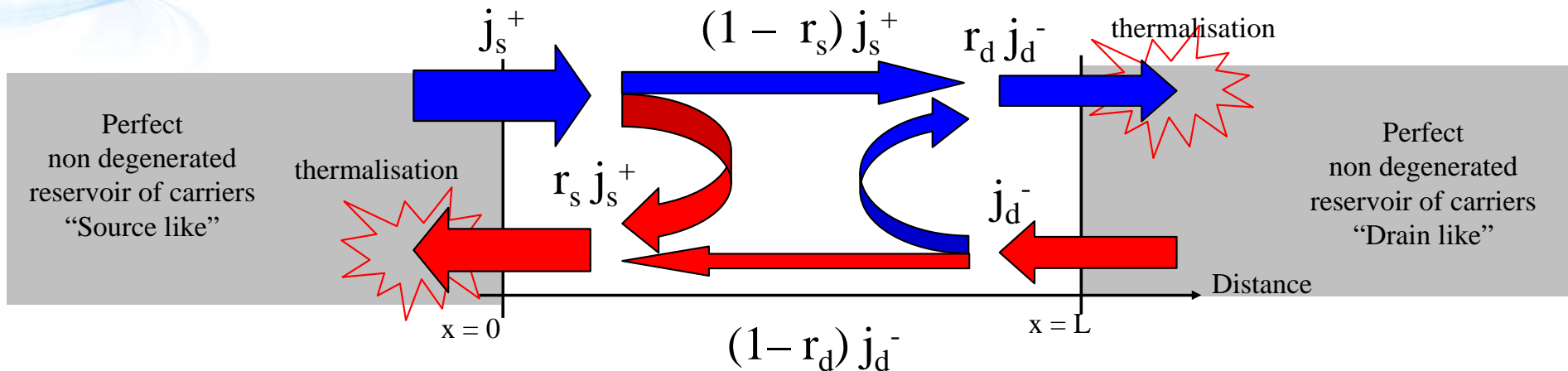


Transistor in ballistic regime

Performance are no longer limited by transport along the channel, but by injection at the source end

Concept of Backscattering Coefficient

Using Mc Kelvey flux theory of Transport :



J. P. McKelvey, R. L. Longini and T. P. Brody, « Alternative approach to the solution of added carrier transport problems in semiconductors », Phys. Rev., vol **123** pp. 51-57 (1961).

Assuming an Inversion Charge Q_i at the virtual source controlled by the gate :
(like in the Natori's model)

$$\frac{I_d}{W} = \frac{(1-r) - \exp(-eV_d/kT)(1-r)}{(1+r) + \exp(-eV_d/kT)(1-r)} Q_i v_{th}$$

$$\frac{I_{d\text{ Sat}}}{W} = \frac{1-r}{1+r} Q_i v_{th}$$

$$\frac{I_{d\text{ Lin}}}{W} = (1-r) \frac{Q_i}{2} v_{th} \frac{eV_d}{kT}$$

A. Rahman and M. S. Lundstrom, « A compact scattering model for the nanoscale double-gate MOSFET », IEEE. TED, vol. 49 p 481 - 489 (2002)



Concept of Backscattering Coefficient

r_{LF} = Back Scattering Coefficient at Low Field (Ohmic Regime)

assuming a constant isotropic mean free path :

$$r_{LF} = \frac{L}{L + \lambda} \quad (\text{using Boltzmann or Fermi Statistics})$$

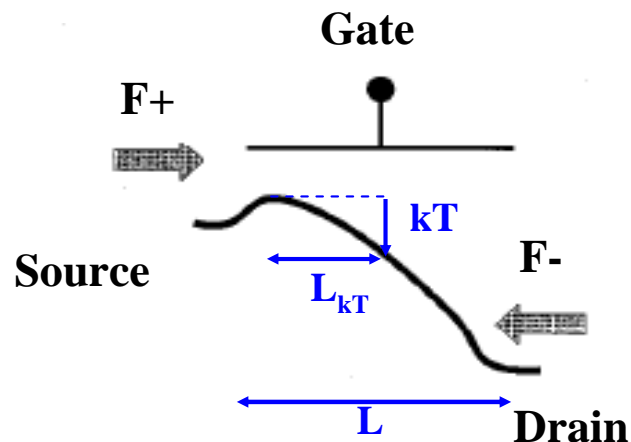
M. Lundstrom, « Fundamentals of Carrier Transport », second edition, Cambridge university press, 2000

When $L \gg \lambda$, **this is consistent with the Drift Diffusion model :**

$$\frac{I_{dLin}}{W} = (1-r) \frac{Q_i}{2} v_{th} \frac{e V_d}{kT} \xrightarrow{L \gg \lambda} \frac{I_{dLinDD}}{W} = \frac{1}{L} \mu_{eff} Q_i V_d \quad \text{if} \quad \lambda = \frac{2 \mu_{eff} kT}{v_{th} e}$$

M. Lundstrom IEEE EDL 22 p. 293 (2001)

r_{HF} = Back Scattering Coefficient at High Field (Saturation Regime)



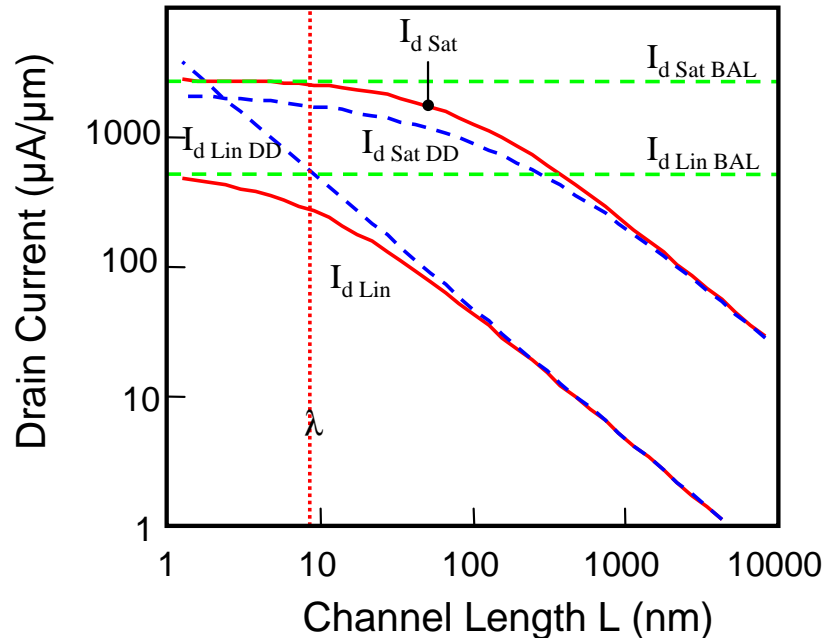
$$r_{HF} = \frac{L_{kT}}{L_{kT} + \lambda} \longrightarrow \text{empirical}$$

intuited as a generalization of r_{LF} $r_{LF} = \frac{L}{L + \lambda}$

M. Lundstrom Z. Ren, IEEE TED 49 p.133 (2002)

Backscattering Coefficient Modeling

In the MOSFET model, we have : $\frac{I_{d\text{ Sat}}}{W} = \frac{1-r}{1+r} Q_i v_{th}$ with $r_{HF} = \frac{L_{kT}}{L_{kT} + \lambda}$



When $L \gg \lambda$, **this is consistent with the Drift Diffusion model :**

$$\frac{I_{d\text{ Sat DD}}}{W} = \frac{1}{L} \mu_{\text{eff}} C_{\text{ox}} \frac{(V_g - V_T)^2}{2} \quad \text{if}$$

$$\lambda = \frac{2 \mu_{\text{eff}} kT}{v_{th} e} \quad (\text{same expression than in ohmic regime})$$

$$\text{and } L_{kT} = L \frac{2 kT / e}{V_{d\text{ SAT}}} = L \frac{2 kT / e}{V_g - V_T}$$

which is in fact **equal to the L_{kT} layer calculated from the Drift Diffusion potential profile !**

$\mu = 200 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$
 $V_{inj} = 1.2 \times 10^5 \text{ m/s}$
 $N_{inv} = 1.45 \times 10^{13} \text{ cm}^{-2}$

- In ohmic regime, Quasi ballistic transport occurs when $L \sim \lambda$
- In saturation, Quasi ballistic transport occurs **when $L_{kT} \sim \lambda$ i.e when $L \sim 10 \lambda$**

M. S. Lundstrom and J. H. Rhee Journal of Computational Electronics, vol. 1 pp 481 - 489 (2002).

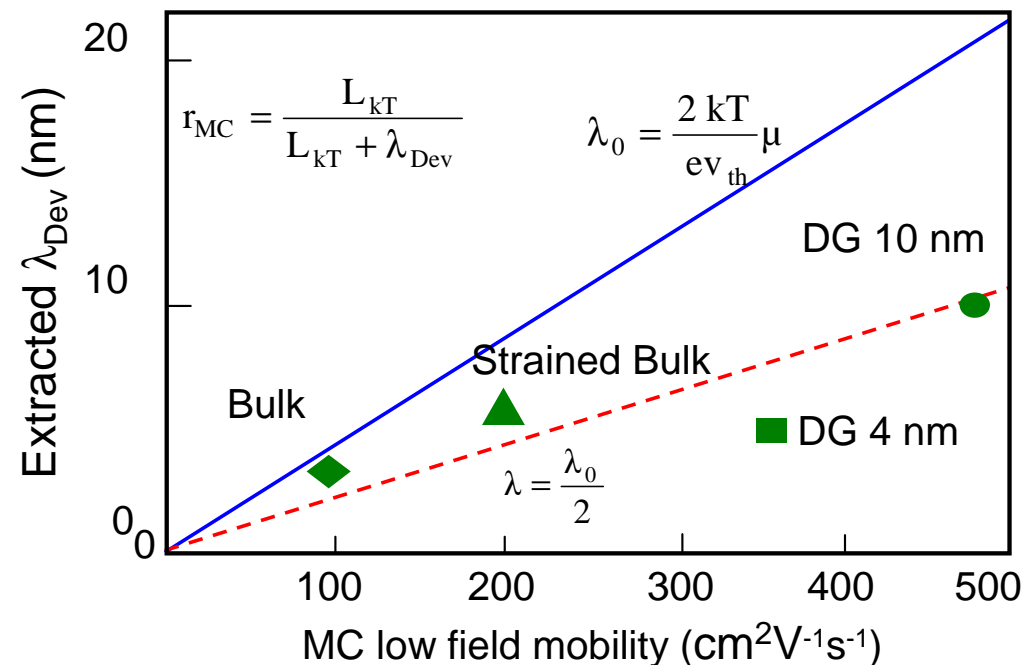
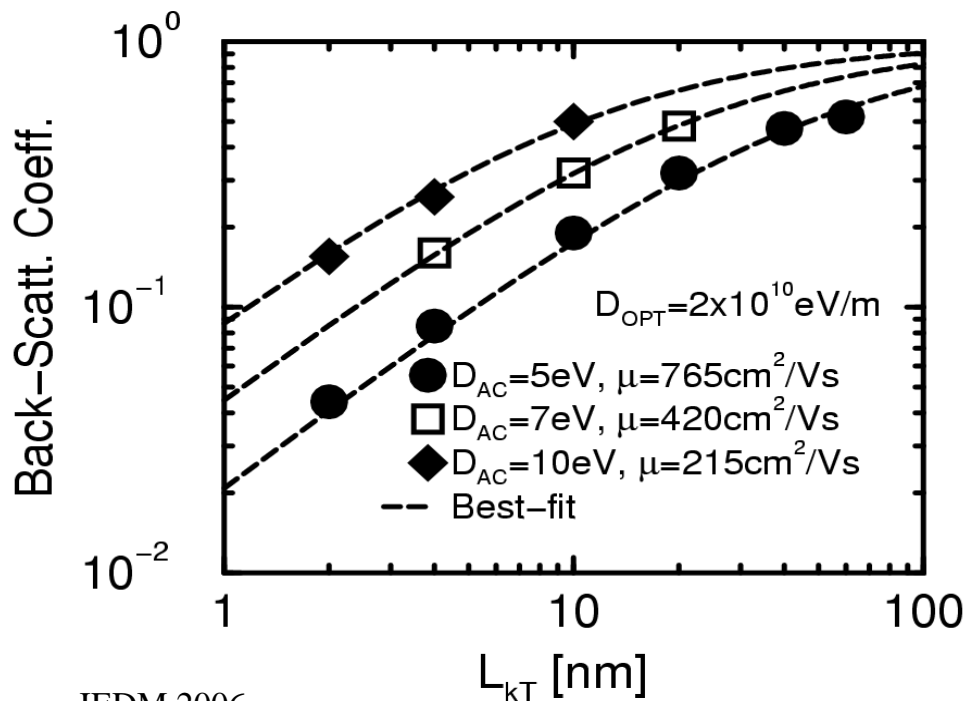
Validation by Monte Carlo Simulation

$$r_{HF} = \frac{L_{kT}}{L_{kT} + \lambda} \quad ??$$

r_{HF} extracted is in fact a function of L_{kT}

$$\lambda_0 = \frac{2 kT}{e v_{th}} \mu \quad ??$$

λ extracted is in fact proportional to μ



IEDM 2006

« Multi Subband Monte Carlo investigation of the mean free path and of the kT layer in degenerated quasi ballistic MOSFETs »

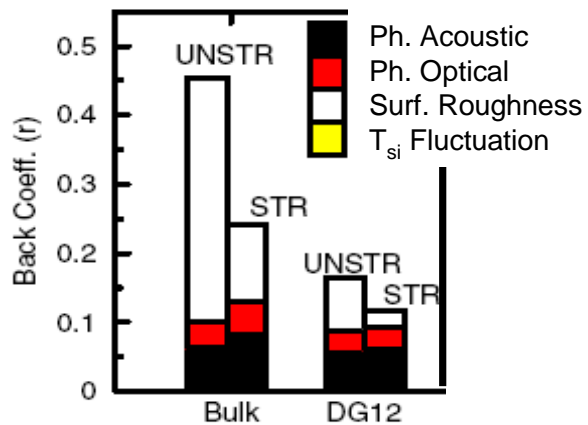
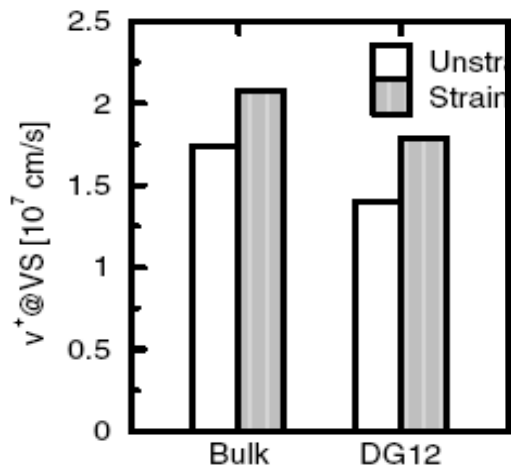
P. Palestri, R. Clerc, D. Esseni, L. Lucci, L. Selmi



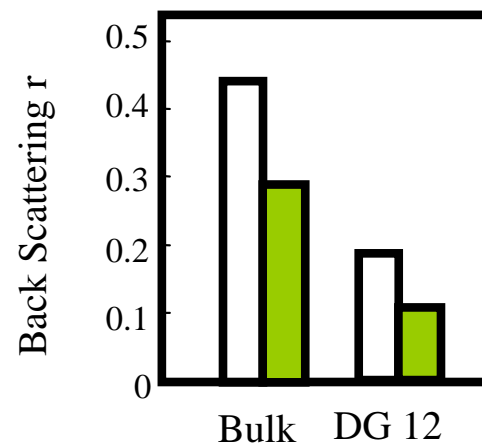
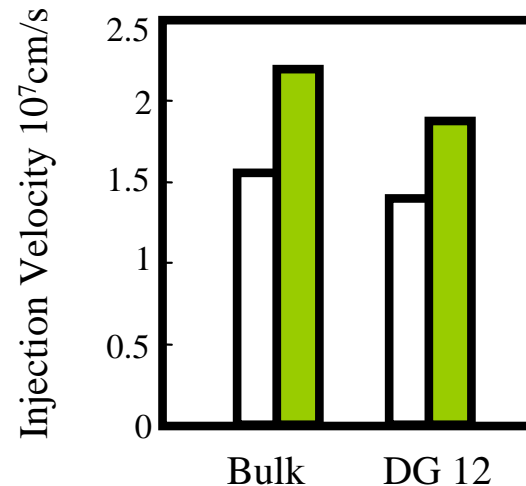
Application to device performance modeling

Strain Silicon : double advantage in QB devices

Multi Subband Monte Carlo Simulations



Analytical Model



$$r = \frac{L_{kT}}{L_{kT} + \lambda}$$

$$\lambda = \frac{1}{2} \times \frac{2 \mu_{\text{eff}} (E_{\text{eff}}) kT/e}{v_{\text{th}}}$$

D. Ponton et.al. Proc. Essdenc 2006 p. 166



Limits of Lundstrom's Model

$$r_{HF} = \frac{L_{kT}}{L_{kT} + \lambda}$$

- is only an empirical formula : **where does it come from ?**
- **How to evaluate L_{kT} ?** ($V(x)$ is not known)

*(In particular the impact of velocity overshoot on $V(x)$,
and thus L_{kT} is not self consistently taken into account)*

$$\lambda = \frac{2 kT/e}{V_{th}} \mu$$

- OK in low field
- **But in high field rather equal to**

$$\lambda = \alpha \frac{2 kT/e}{V_{th}} \mu \quad \text{with} \quad \alpha \approx \frac{1}{2}$$

V_{th} and r_{HF} are valid in **non degenerated inversion layers**

- How to generalize L_{kT} and λ in degenerated inversion layer ?

(An increase of V_{inj} may also degrade r : is subband engineering a viable strategy ?)



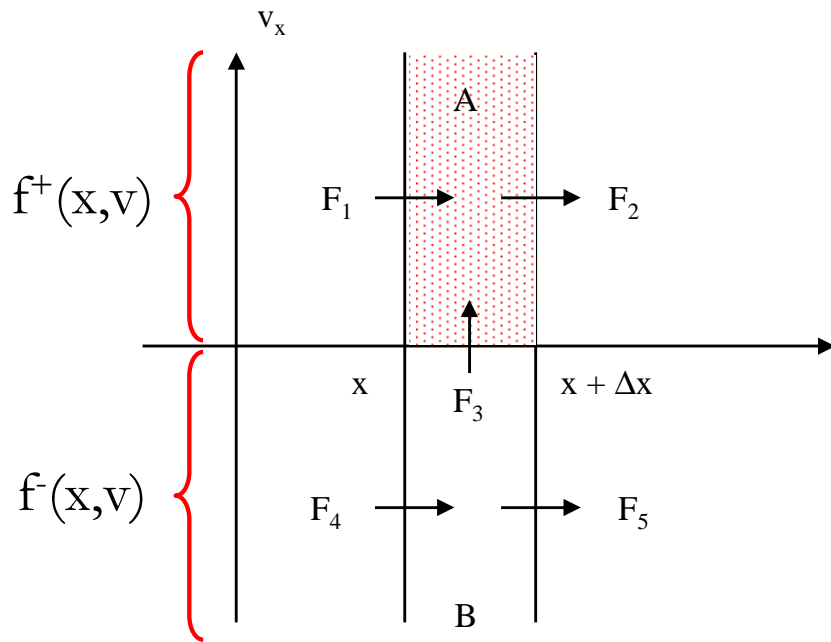
Beyond Lundstrom's Model

R. Clerc, P. Palestri, L. Selmi

« On the Physical Understanding of the kT -Layer Concept in Quasi-Ballistic Regime of Transport in Nanoscale Devices »
IEEE TED 53, p 1634 – 1640 (2006)

1D flux conservation in the relaxation length approximation

1D Balance Equation in the phase space



$$F_1 + F_3 = F_2 + C_{A \rightarrow B}$$

$$F_1 = \int_0^{\infty} f(x, v_x) v_x dv_x = \Phi^+(x)$$

Collision Integral (**relaxation length approximation**) :

$$C_{A \rightarrow B} = \Delta x \int_0^{\infty} \frac{f(x, v_x) - f_{LE}(x, v_x)}{\lambda / v_x} dv_x = \Delta x \frac{\Phi^+(x) - \Phi_{LE}^+(x)}{\lambda}$$

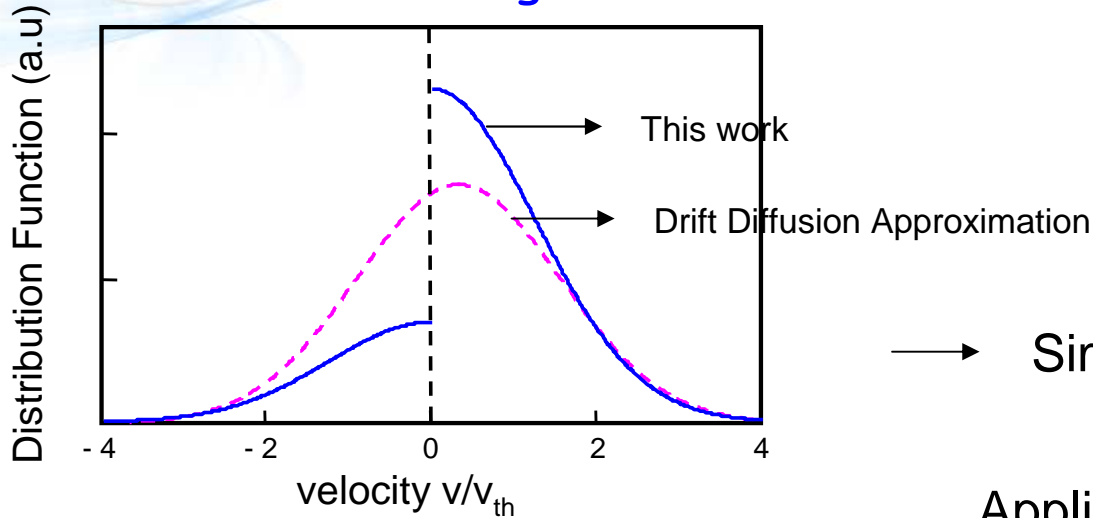
Balance Equation for the flux of carrier along the device

$$\frac{d\Phi^-}{dx} = -f(x, 0^-) \frac{eF}{m} - \frac{\Phi^+(x) - \Phi_{LE}^+(x)}{\lambda}$$

→ Assumption on the shape of $f(x, v_x)$ needed

The quasi ballistic Drift Diffusion

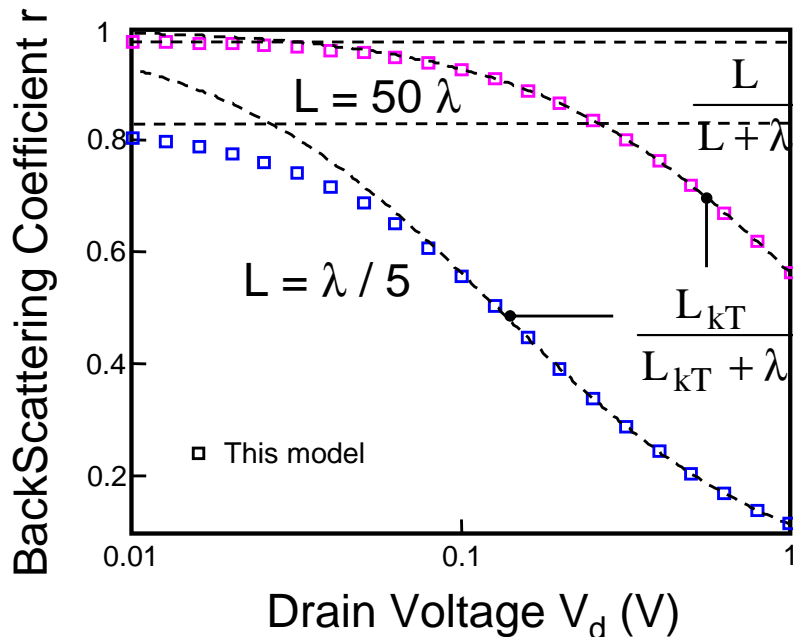
Assuming local Maxwellian distribution function (at each point) :



$$f^{\pm}(x, v) = 2 \sqrt{\frac{m}{2 \pi kT}} n^{\pm}(x) \exp\left(-\frac{m v^2}{2 kT}\right)$$

→ Similar to Drift Diffusion

Application to the backscattering coefficient calculation:



$$r = \frac{L_{kT} (1 - \beta)}{\lambda + L_{kT} (1 - \beta)}$$

$$\beta = \exp(-L / L_{kT})$$

This equation includes both :

$$r_{LF} = \frac{L}{\lambda + L}$$

$$r_{HF} = \frac{L_{kT}}{\lambda + L_{kT}}$$



The quasi ballistic Drift Diffusion

We thus have now a new formalism which includes :

Thermal velocity limitation at the source (Ballistic limit)

Same backscattering coefficient in high and low field (Ballistic mobility)

With no need to calculate LkT ...

→ **An alternative to Drift Diffusion ?**

Using this approach, if we derive the current flux :

$$\Phi = q (n^+ - n^-) v_{th} = -\lambda_x v_{th} \frac{dn}{dx} - \frac{\lambda_x v_{th}}{kT/q} 2n^- F_x$$

Using : $n = n^+ + n^-$

$$\Phi = (n^+ - n^-) v_{th} = -D' \frac{dn}{dx} - n \mu' F_x$$

$$\mu' = \frac{\mu}{1 + \mu F_x / v_{th}}$$

Except for the boundary conditions, it is equivalent to Drift Diffusion,
including saturation velocity !

Idem for H. Wang, G. Gildenblat,

“Scattering matrix based compact MOSFET model”, in IEDM Tech. Dig., pp. 125–128 (2002)

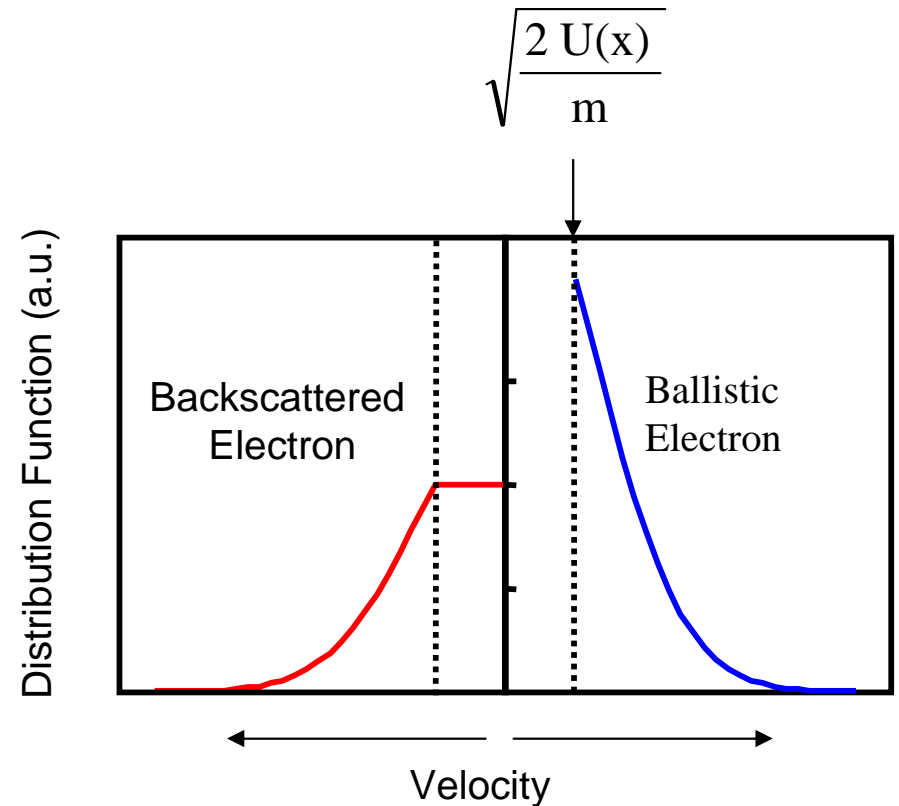
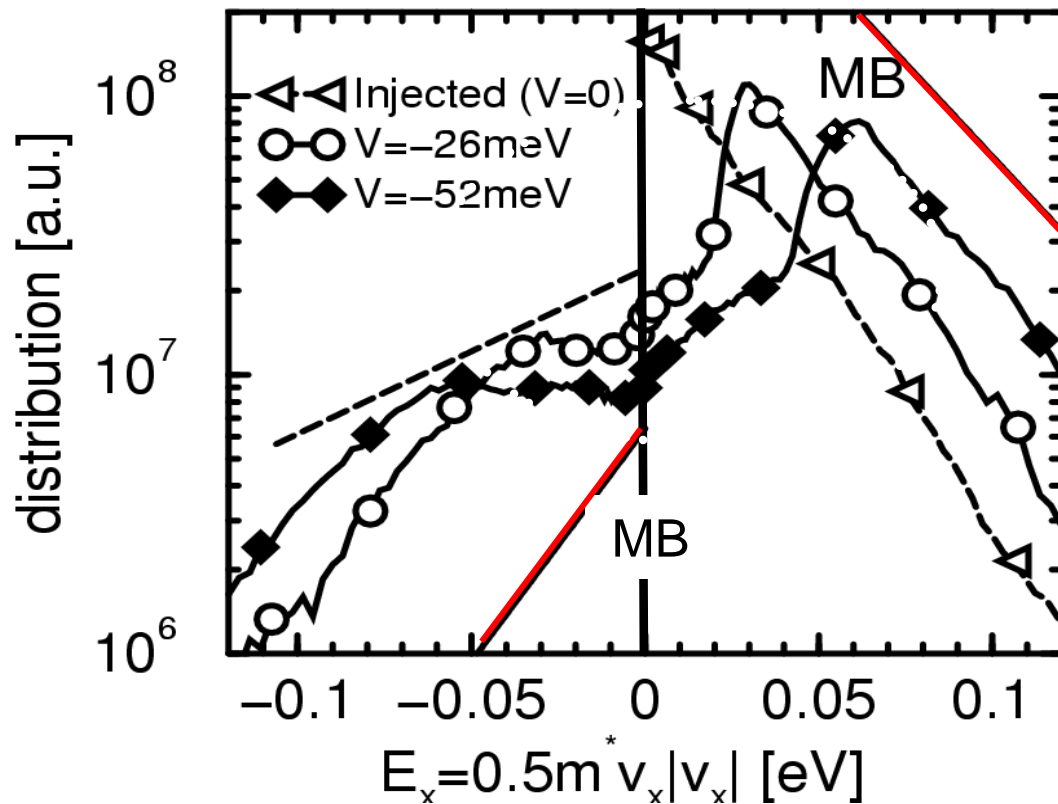
Non Thermal Approach

What is wrong with quasi ballistic Drift Diffusion ?

$$f^\pm(x, v) = 2 \sqrt{\frac{m}{2\pi kT}} n^\pm(x) \exp\left(-\frac{m v^2}{2 kT}\right)$$

A more suitable approximated distribution function (at each point) :

From MC simulations

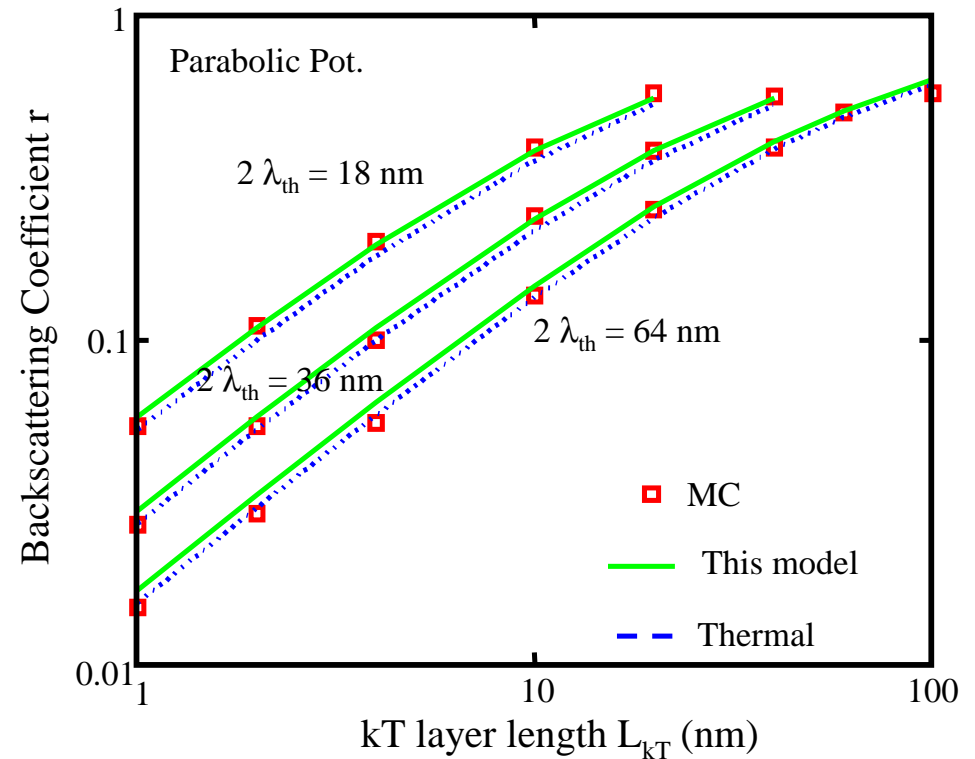
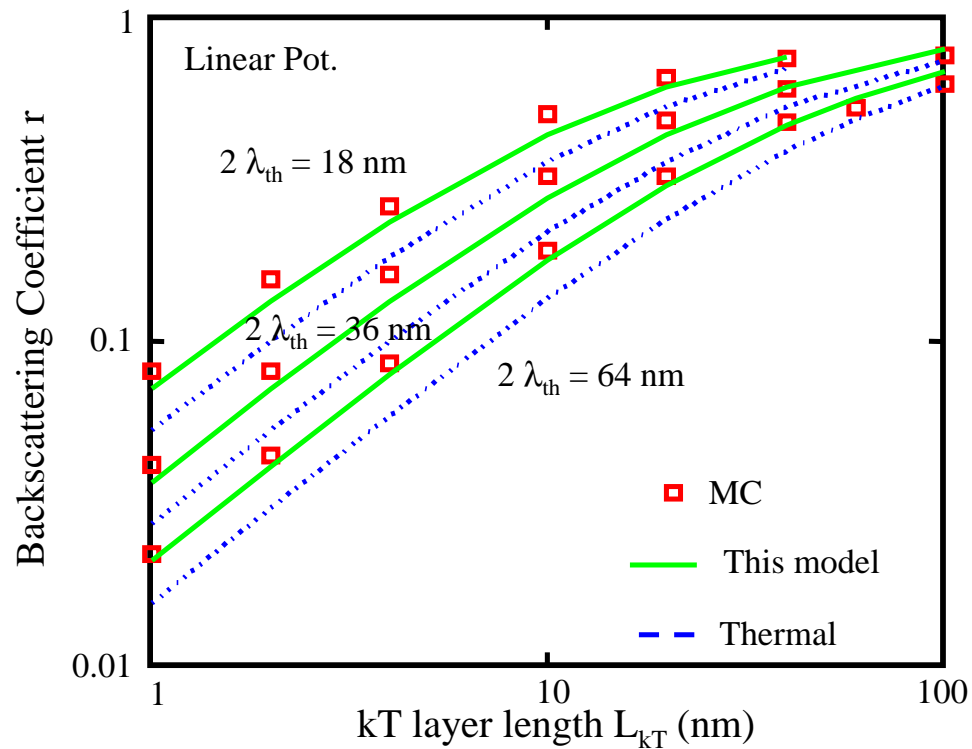


Non Thermal Approach

Backscattering formula :

$$r = \frac{L_{kT} A(L_{kT}, L)}{2\lambda + L_{kT} A(L_{kT}, L)}$$

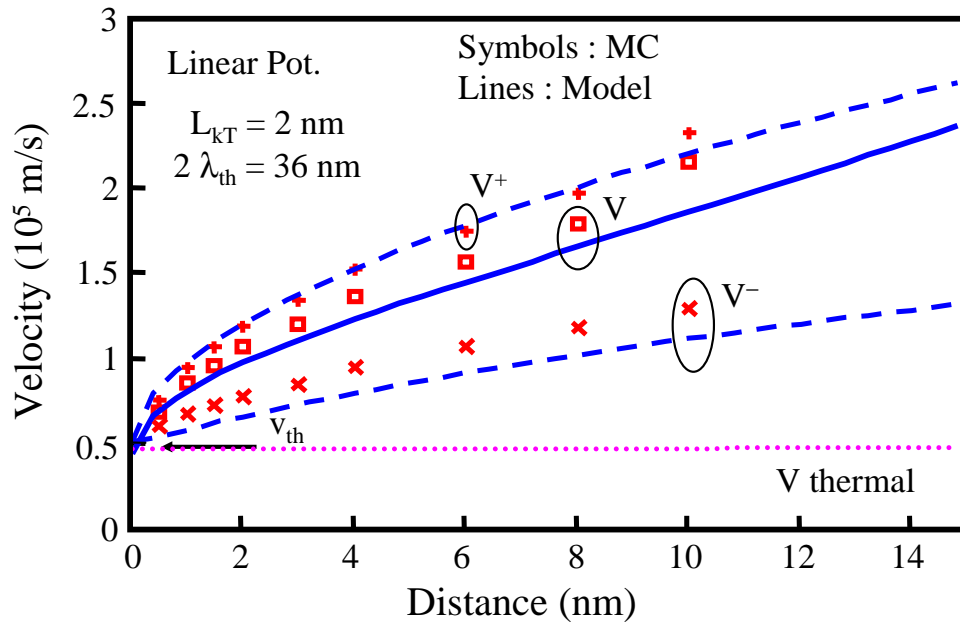
$$A(L_{kT}, L) = \int_0^{L_{kT}/L} \left[\frac{2\lambda(u)}{2\lambda} \left(1 + \frac{U(u)}{kT}\right) \right]^{-1} du$$



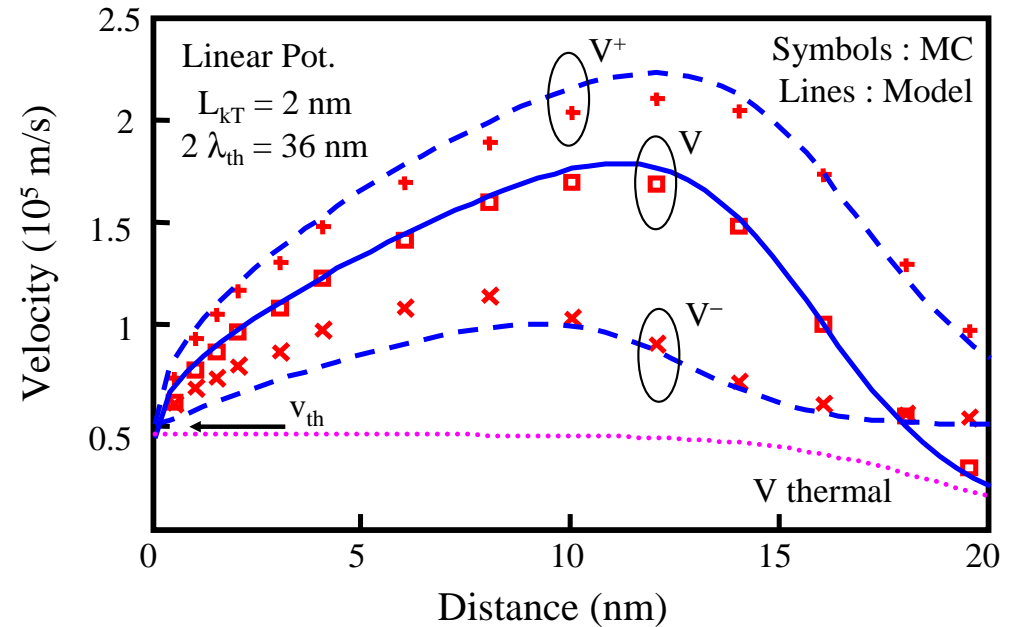
Non Thermal Approach

Velocity profile in high field in a non self consistent linear potential profile

With absorbing drain :



With real drain (emitting) :





Ballisticity extraction from experiments

Ballisticity extraction from experiments

Is there any experimental confirmation of the quasi ballistic nature of transport ?

Not clear yet !

Let us show some results :

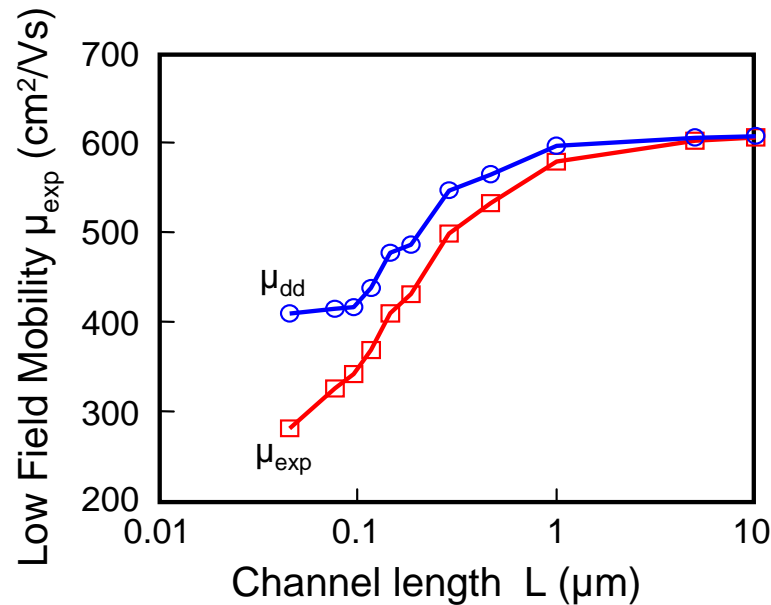
- let us consider only the ohmic regime
(more simple : you do not need to know the charge at the virtual source)
- a standard low field mobility extraction has been performed on :
65 like nm Bulk and undoped Fully Depleted SOI technology
featuring physical gate length down to 40 nm
- according quasi ballistic theory :

$$\frac{I_{d \text{ Lin}}}{W} = (1-r) \frac{Q_i}{2} v_{th} \frac{e V_d}{kT} = \frac{1}{L} \mu_{app} Q_i V_d \longrightarrow \mu_{app} = \frac{L}{L + \lambda} \mu_{dd}$$

Apparent mobility should decrease with L in the quasi ballistic regime

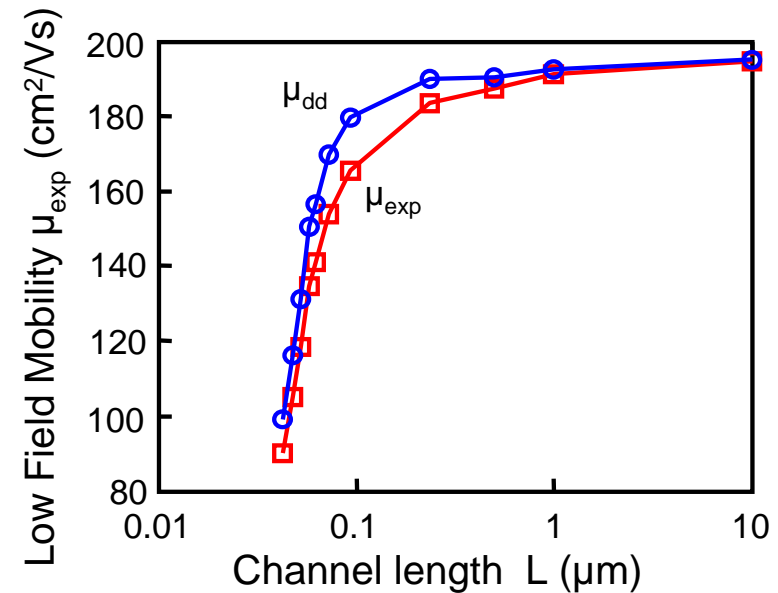
Ballisticity extraction from experiments

Bulk nMOS



Bulk 65nm CMOS technology
 doped channel ($\approx 10^{17}/\text{cm}^3$) with halos,
 SiON gate oxide (CET=2.2nm)
 polysilicon gate.

Undoped FD-SOI nMOS



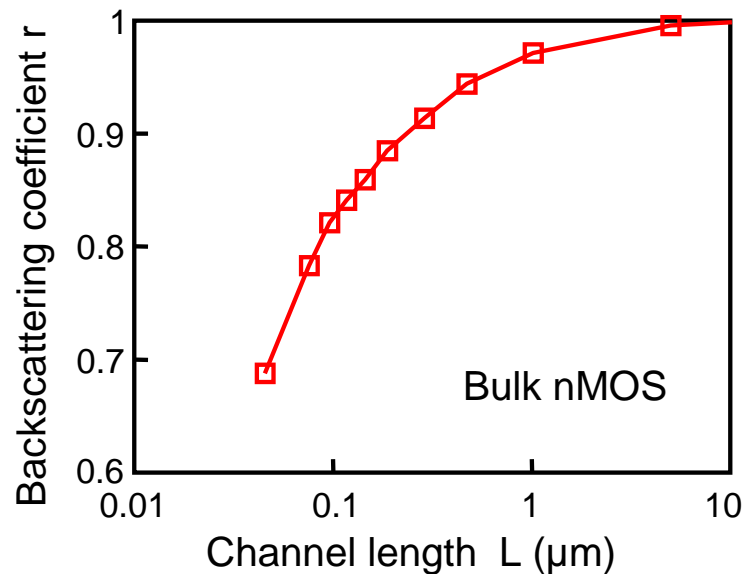
Undoped Fully depleted SOI
 Body thickness of 10 nm
 Metal gate TiN, 2.5 nm of HfSiON dielectric
 Raised source and drain

Ballisticity extraction from experiments

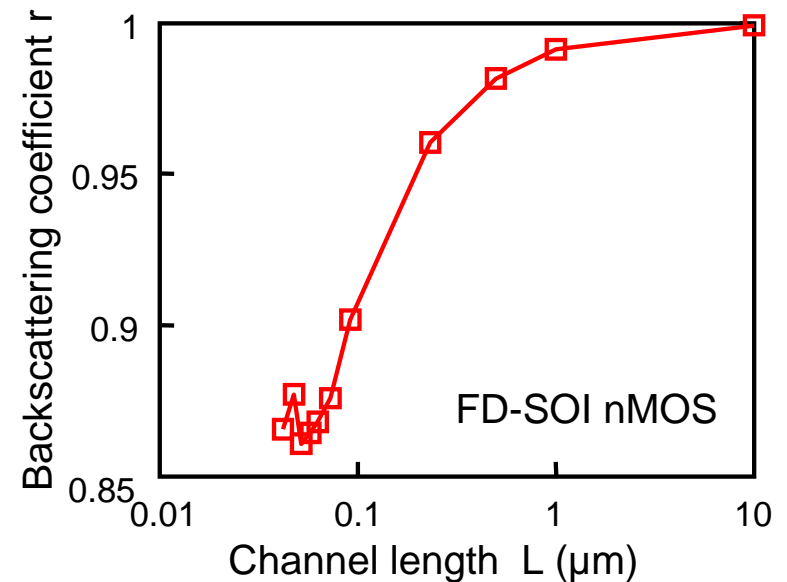
The backscattering coefficient has been extracted using

$$r = 1 - \frac{\mu_{app}}{\mu_{bal}}$$

Bulk nMOS $r_{LF} \sim 0.7$



Undoped FD-SOI nMOS $r_{LF} \sim 0.85$



Extracted value of r are very large ... Neutral defects ?



Conclusions

Conclusions

The Lundstrom backscattering theory provides **a powerful guideline** to analyse **qualitatively** device performance in the quasi ballistic regime

However, **It is not a compact model** (How to compute LkT ?)

If you try to generalize this approach along the channel

→ quasi ballistic drift diffusion

→ which is **similar to drift diffusion with saturation velocity in high field regime**

You need to account for the ballistic distribution of carriers

However, experiments shows a more complex picture :

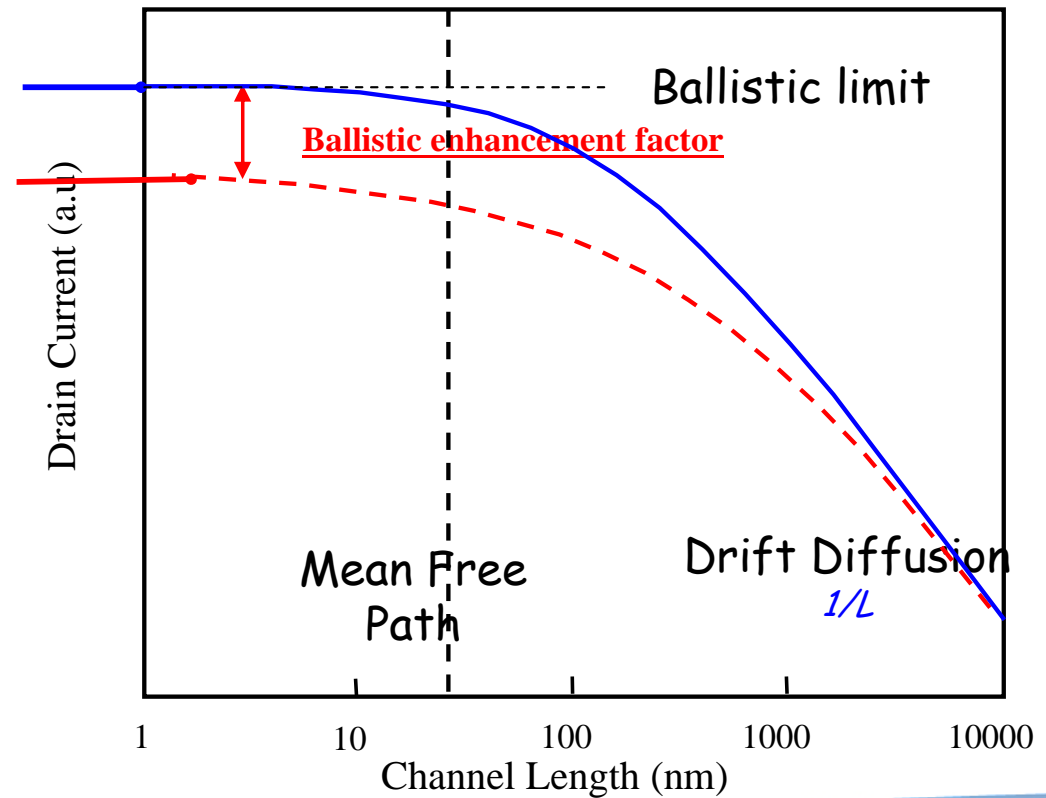
we are not sure yet to operate in the quasi ballistic regime (neutral defects ?)



Thank you for your attention !

More Exact Quasi Ballistic Model

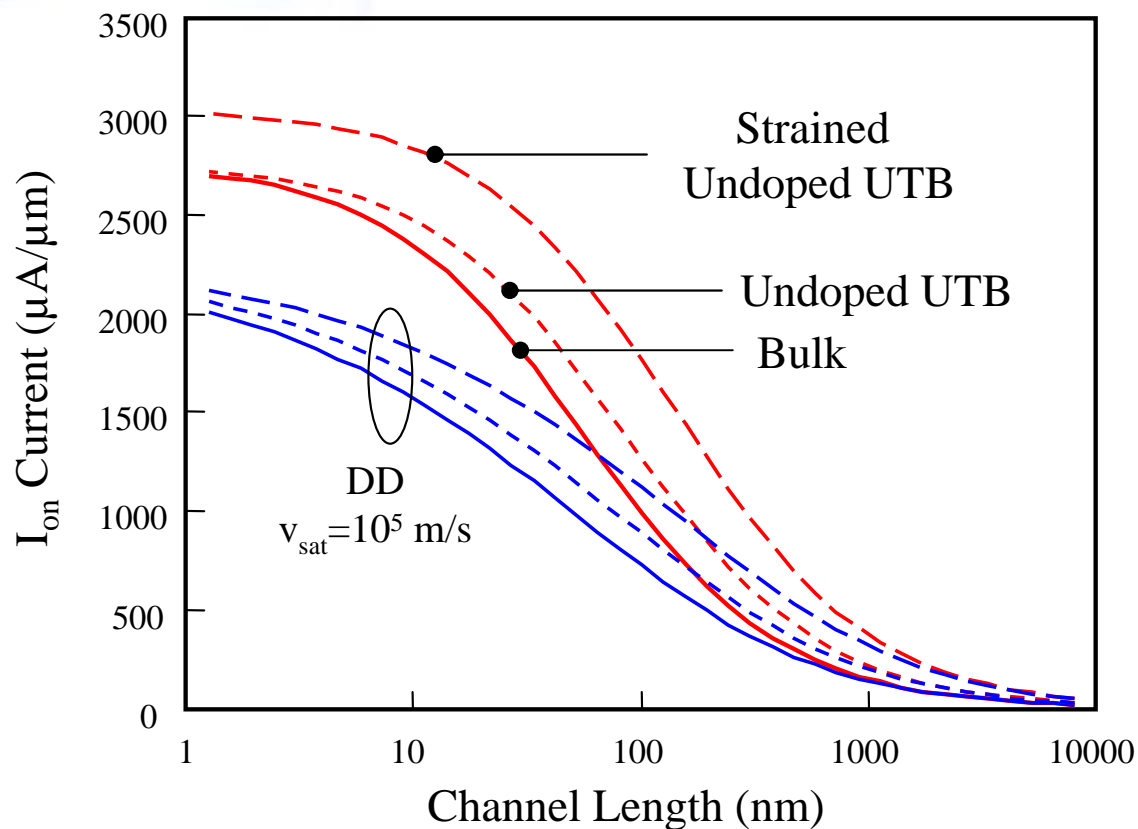
Drift Diffusion Analytical Model
(Velocity Saturation)





Application to device performance enhancement

Device Optimisation in the Quasi Ballistic Regime



BULK
 $\mu = 130 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$
 $V_{inj} = 1.2 \times 10^5 \text{ m/s}$
 $N_{inv} = 1.45 \times 10^{13} \text{ cm}^{-2}$

Undoped UTB
 $\mu = 200 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$
 $V_{inj} = 1.2 \times 10^5 \text{ m/s}$
 $N_{inv} = 1.45 \times 10^{13} \text{ cm}^{-2}$

Strained Undoped UTB
 $\mu = 370 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$
 $V_{inj} = 1.3 \times 10^5 \text{ m/s}$
 $N_{inv} = 1.45 \times 10^{13} \text{ cm}^{-2}$

2 possible strategies to improve I_{on} :

- **improving λ** , which mean **improving μ**
 = effective field mobility

like in pure Drift Diffusion model !

- **improving V_{inj}**
(subband engineering)

by DOS reduction

→ Still no clear experimental evidence



Re-investigation of the kT layer Concept

$$r_{\text{HF}} = \frac{L_{\text{kT}}}{L_{\text{kT}} + \lambda}$$

has been derived using Quasi Ballistic Drift Diffusion.

key assumptions on which this formula is based :

- Boltzmann statistics in the contacts
- Non Self consistent potential
- use of the relaxation length approximation with a constant λ
 - λ should be energy dependent (especially at high field)
- the population of backscattered carriers (f^-) has an equilibrium Maxwellian distribution :
 - at each point x ,
 - regardless of the channel length
 - and of the magnitude of the electric field
 - reasonable approximation
 - Only close to the virtual source*
 - Or along the channel when many collisions are involved*