

## IMPROVEMENTS TO $\alpha$ -Si RPI-TFT MODEL: NOW EXTRINSIC AND WITH CORRECT ACCOUNT OF THE POSITIVE DIFFERENTIAL CONDUCTIVITY AFTER SATURATION

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We introduce an improved TFT model based on the RPI compact model. The new model accurately describes the output conductance in the entire range of drain and gate bias voltages. We revise the approach based on the channel-length modulation parameter  $\lambda_0$  to account for positive differential conductivity after saturation. The traditional approach suffers from non-monotonic behavior of the differential conductivity with an increasing drain-to-source bias at small bias values. Our new approach provides for the correct monotonic decrease of the differential conductivity. This is done by first introducing a new asymptotic "after saturation" equation for the drain current. This equation was derived from the condition, that asymptote should be tangential to the output characteristic before saturation in Charge-Sheet Model (CSM). Second, we use a Hölder mean with exponent  $-m$  of the linear part of the CSM output characteristic before saturation and the new equation for drain current after saturation. Note that instead of using here new asymptotic "after saturation" equation we need to modify one as discussed in our presentation. We have transformed the RPI model into an extrinsic analytical model that accounts for the source and drain series resistances. The suggested improvements could be useful for most FETs compact models used in modern compact circuit simulators.

## CHARGE-SHEET-MODEL

Intrinsic model:  $R_S = R_D = 0$

Saturation current and voltage:

$$I_{sat} = \alpha \beta V_{GT}^2 / 2$$

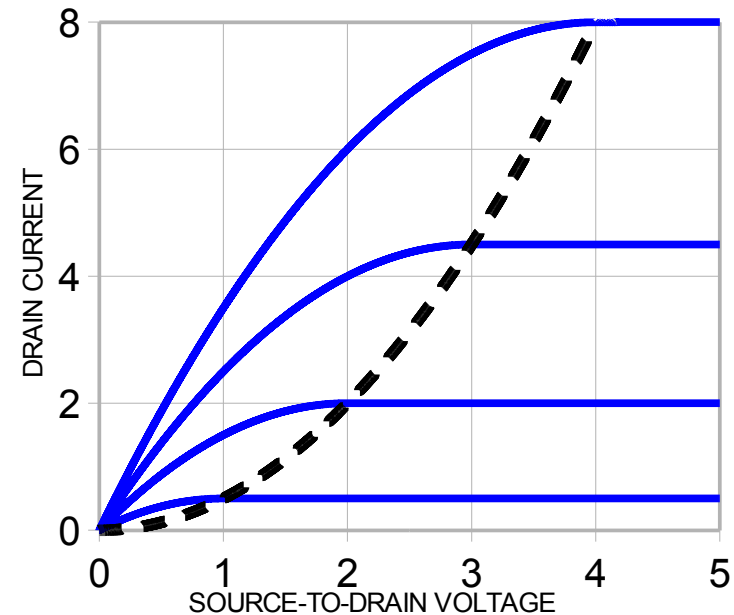
$$V_{SAT} = \alpha V_{GT}$$

Current-voltage characteristics:

$$I = \begin{cases} \beta (V_{GT} V_{DS} - V_{DS}^2 / 2\alpha), & V_{DS} < V_{SAT} \\ I_{sat}, & V_{DS} \geq V_{SAT} \end{cases}$$

$$I_{dem} = \beta V_L^2 / \alpha$$

$$V_{dem} = V_L$$



We use dimensionless units:

## ACCOUNT FOR DRIFT VELOCITY SATURATION

Model for drift velocity:

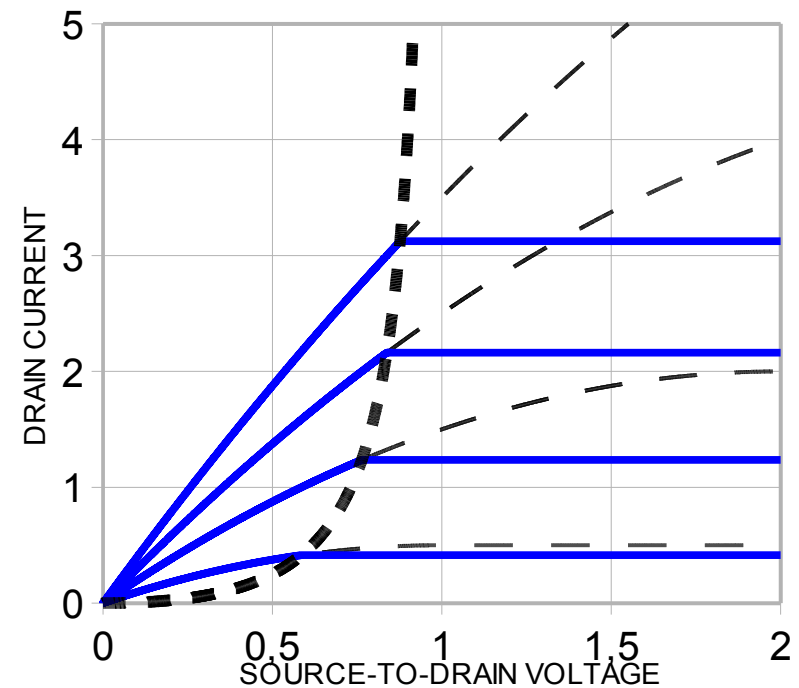
$$v = \begin{cases} \mu E, & E < E_s \\ v_s, & E > E_s \end{cases}$$

Saturation current:

$$I_{sat} = \frac{\beta V_L^2}{\alpha} \left( \sqrt{1 + \left( \frac{\alpha V_{GT}}{V_L} \right)^2} - 1 \right)$$

Saturation voltage:

$$V_{SAT} = V_L \left[ \left( 1 + \frac{\alpha V_{GT}}{V_L} \right) - \sqrt{1 + \left( \frac{\alpha V_{GT}}{V_L} \right)^2} \right]$$



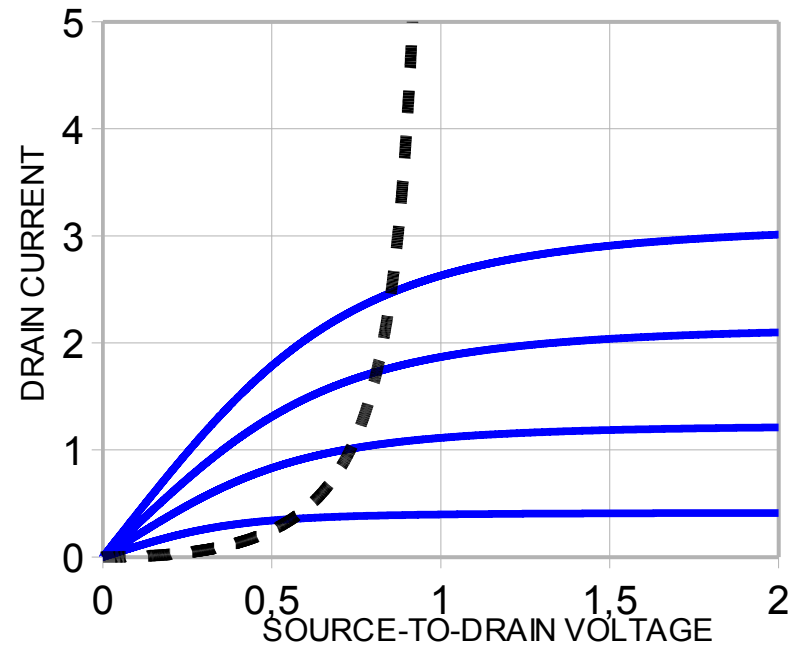
## USING THE HÖLDER MEAN FOR SMOOTH TRANSITION BETWEEN THE DRAIN CURRENT BEFORE AND AFTER SATURATION

Interpolation, based on a Hölder mean of power  $-m$ , gives smooth transition between linear and saturation current:

$$I = \frac{I_{1L} I_{sat}}{\left[ I_{1L}^m + I_{sat}^m \right]^{1/m}}$$

Here:

$$I_{1L} = \beta V_{GT} V_{DS} = a V_{DS}$$



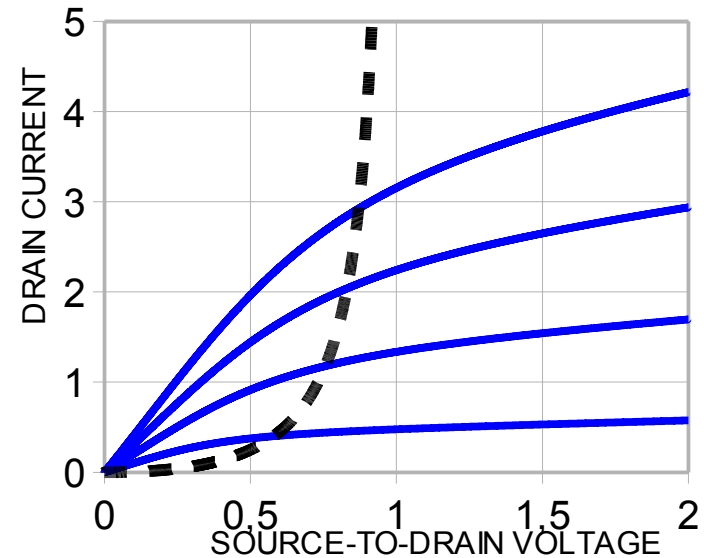
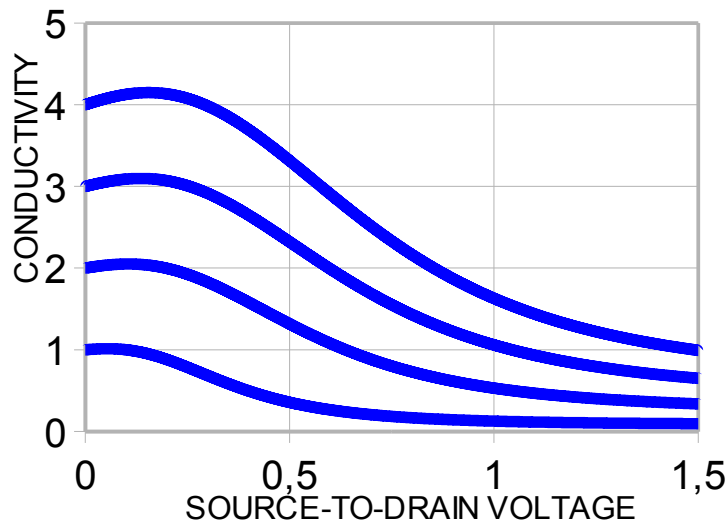
## TRADITIONAL APPROACH TO ACCOUNT FOR THE POSITIVE DIFFERENTIAL CONDUCTIVITY AFTER SATURATION DUE TO CLM AND DIBL EFFECTS

The traditional approach to account for positive differential conductivity after saturation based on the channel-length modulation parameter  $\lambda_0$

$$I = \frac{I_{1L} I_{sat}}{\left[ I_{1L}^m + I_{sat}^m \right]^{1/m}} \times \left( 1 + \lambda_0 V_{DS} \right)$$

Asymptotic "after saturation" equation for the drain current:

$$I_2 = I_{sat} \left( 1 + \lambda_0 V_{DS} \right) = c + b V_{DS}$$



⇐ However, the traditional approach suffers from non-monotonic behavior of the differential conductivity with an increasing drain-to-source bias at small bias values:

$$I \approx \beta V_{GT} \left( V_{DS} + \lambda_0 V_{DS}^2 \right)$$

## NEW ASYMPTOTIC "AFTER SATURATION" EQUATION FOR THE DRAIN CURRENT

We introduce a new asymptotic "after saturation" equation for the drain current:

$$I_2^* = I^* + \lambda_0 I_{sat} (V_{DS} - V^*) = c^* + b V_{DS}$$

Parameters  $I^*$  and  $V^*$  are derived from the condition, that new asymptote should be tangential to the output characteristic before saturation in Charge-Sheet Model:

$$g_1 = \frac{dI_1}{dV_{DS}} = \beta \left( V_{GT} - \frac{1}{\alpha} V_{DS} \right)$$

$$g_2 = \frac{dI_2^*}{dV_{DS}} = \lambda_0 I_{sat}$$

$$g_1(V_{DS} = V^*) = g_2 \quad \Rightarrow$$

$$\begin{cases} V^* = \alpha \left( V_{GT} - \frac{\lambda_0}{\beta} I_{sat} \right) \\ I^* = \frac{\alpha}{2} \left( \beta V_{GT}^2 - \frac{\lambda_0^2}{\beta} I_{sat}^2 \right) \end{cases}$$

$$\text{If } g_1(V_{DS} = V_{sat}) > g_2 \quad \Rightarrow$$

$$V^* = V_{sat}, I^* = I_{sat}$$

$$\text{If } g_1(V_{DS} = 0) < g_2 \quad \Rightarrow$$

$$V^* = 0, I^* = 0$$

## MODIFICATION OF THE NEW ASYMPTOTIC “AFTER SATURATION” EQUATION BEFORE USING ONE IN THE HÖLDER MEAN EQUATION

For a smooth transition between the drain current before and after saturation we use a Hölder mean with exponent  $-m$  of the  $I_{1L}$  and  $I_2^{**}$

$$I = \frac{I_{1L} I_2^{**}}{\left[ I_{1L}^m + I_2^{**m} \right]^{1/m}}$$

To obtain the right asymptotic slop in this equation after saturation, instead of using here  $I_2^*$  we need to use  $I_2^{**}$ :

$$I_2^{**} = c^{**} + b^{**} V_{DS}$$

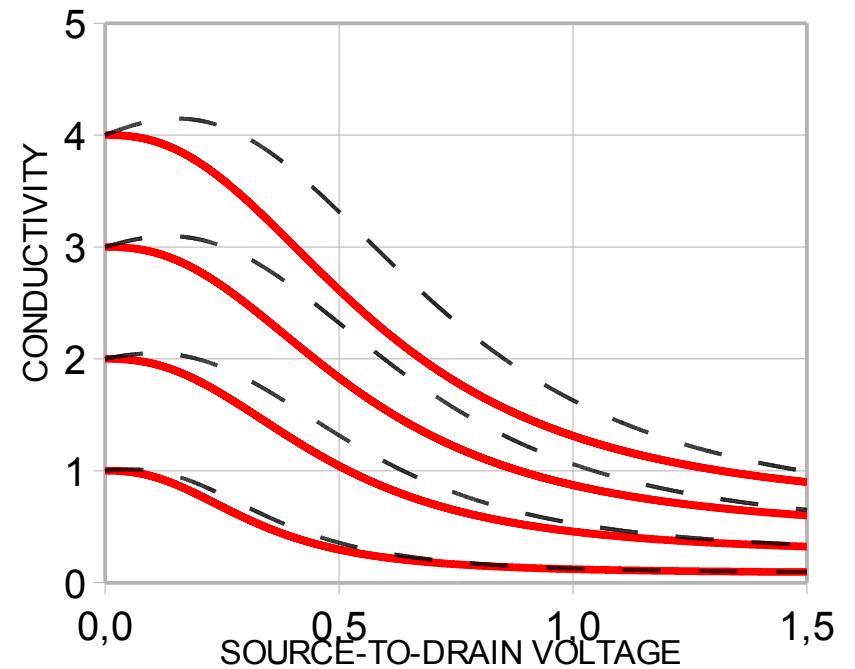
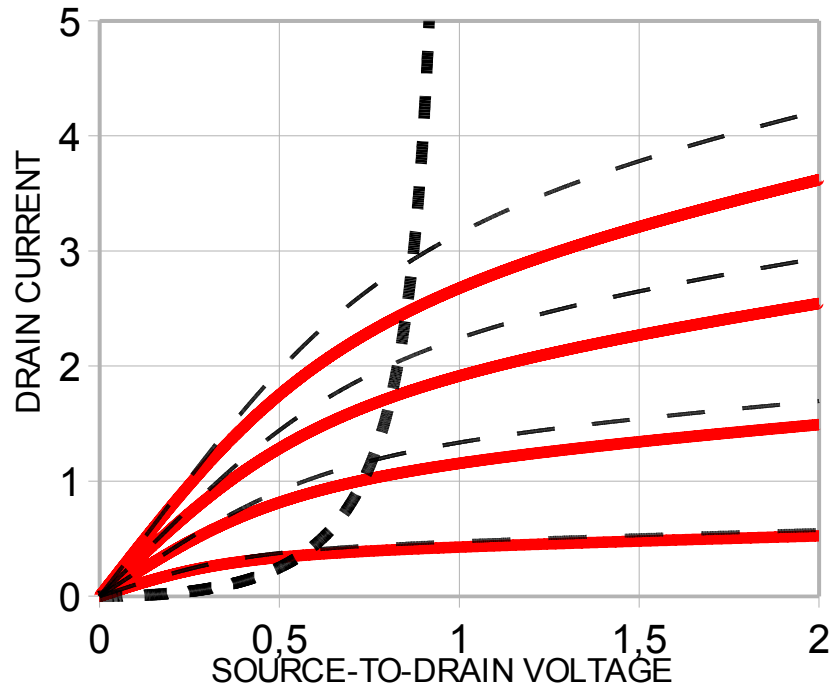
with

$$b^{**} = \frac{a}{\left[ a^m - b^{*m} \right]^{1/m}} b \qquad c^{**} = \frac{a}{\left[ a^m - b^{*m} \right]^{1/m}} c^*$$

We can rewrite equation for the drain current :

$$I = \frac{I_{1L} I_2^*}{\left[ I_{1L}^m + I_2^{*m} - (I_2^* - I^* + \lambda_0 I_{sat} V^*) \right]^{1/m}}$$

## COMPARISON OF THE RPI MODEL BEFORE AND AFTER IMPROVEMENTS



Dotted lines are for the  $\alpha$ -Si RPI-TFT model before improvements and red lines for improved model. The thick dotted line for the dependence of the saturation current on drain-to-source bias.

**TRANSFORMATION OF THE RPI MODEL INTO  
EXTRINSIC ANALYTICAL COMPACT MODEL (1)  
LINEAR MODE**

$$\begin{cases} V_{ds} = V_{DS} + I R_T \\ V_{gt} = V_{GT} + I R_S \end{cases} \quad R_T = R_S + R_D$$

$$g_{chi} = \frac{1}{\beta V_{gt}} \quad 1/g_{ch0} = R_T + 1/g_{chi} \quad g_{ch0} = \frac{\beta V_{gt}}{1 + \beta V_{gt} R_T}$$

$$I_{1L} = \frac{1 + \beta(V_{gt} R_T + V_{ds} R_S)}{2\beta R_S R_T} \left( 1 - \sqrt{1 - \frac{4\beta^2 R_S R_T V_{gt} V_{ds}}{(1 + \beta(V_{gt} R_T + V_{ds} R_S))^2}} \right)$$

$$I_{1L}(V_{ds} \rightarrow \infty) \rightarrow \frac{V_{gt}}{R_S}$$

**TRANSFORMATION OF THE RPI MODEL INTO  
EXTRINSIC ANALYTICAL COMPACT MODEL (2)  
SATURATION**

$$I_{sat} = \frac{\alpha \beta V_{gt}^2}{1 + \alpha \beta R_S V_{gt} + \sqrt{1 + 2 \alpha \beta R_S V_{gt} + (\alpha V_{gt} / V_L)^2}}$$

$$V_{sat} = I_{sat} R_T + \alpha \left( V_{gt} - I_{sat} (R_S + 1 / \beta V_L) \right)$$

**TRANSFORMATION OF THE RPI MODEL INTO  
EXTRINSIC ANALYTICAL COMPACT MODEL (3)  
AFTER SATURATION**

If  $I^* < I_{sat}$  we have  $g_2(I^*, V^*) = \lambda_0 I_{sat} > g_1(I_{sat}, V_{sat})$

$$I_2^* = \left(1 + \alpha \beta R_S V_{gt} - \lambda_0 (\alpha R_S - R_T) I_{sat}\right) / \alpha \beta R_S^2 \times$$

$$\left\{ 1 - \sqrt{1 - \frac{\alpha \beta R_S^2 \left( 2 \lambda_0 (V_{ds} - \alpha V_{gt}) I_{sat} + \alpha \left( \beta V_{gt}^2 + \frac{\lambda_0^2}{\beta} I_{sat}^2 \right) \right)}{\left(1 + \alpha \beta R_S V_{gt} - \lambda_0 (\alpha R_S - R_T) I_{sat}\right)^2}} \right\}$$

If  $I^* > I_{sat}$  we have  $g_2(I^*, V^*) = \lambda_0 I_{sat} < g_1(I_{sat}, V_{sat})$

$$I_2^* = I_{sat} \frac{1 + \lambda_0 (V_{ds} - \alpha V_{gt}) + \frac{\lambda_0 \alpha}{\beta V_L} I_{sat}}{1 + \lambda_0 I_{sat} (R_T - \alpha R_S)}$$

**TRANSFORMATION OF THE RPI MODEL INTO  
EXTRINSIC ANALYTICAL COMPACT MODEL (4)  
THE EQUATION FOR DRAIN CURRENT**

$$I^* = \frac{\alpha \beta R_S V_{gt} + 1}{\alpha \beta R_S^2} \left( 1 - \sqrt{1 - \frac{2 \alpha \beta R_S^2 \left( \frac{\alpha \beta}{2} V_{gt}^2 - \frac{\alpha \lambda_0^2}{2 \beta} I_{sat}^2 \right)}{(\alpha \beta R_S V_{gt} + 1)^2}} \right)$$

$$V^* = \alpha \left( (V_{gt} - I R_S) - \frac{\lambda_0}{\beta} I_{sat} \right)$$

$$I = \frac{I_{1L} I_2^*}{\left[ I_{1L}^m + I_2^{*m} - (I_2^* - I^* + \lambda_0 I_{sat} V^*) \right]^{1/m}}$$

## CONCLUSIONS

- We introduce an improved TFT model based on the  $\alpha$ -Si RPI-TFT compact model
- We revise the approach based on the channel-length modulation parameter  $\lambda_0$  to account for positive differential conductivity after saturation. The traditional approach suffers from non-monotonic behavior of the differential conductivity with an increasing drain-to-source bias at small bias values. Our new approach provides for the correct monotonic decrease of the differential conductivity
- In addition, we have transformed the RPI model, that is intrinsic initially, into an extrinsic analytical compact model that accounts for the source and drain series resistances. The extrinsic compact model is useful to analyze and predict the behaviour of  $\alpha$ -Si TFTs and for the computer-aided design of integrated circuits, since it can improve convergence and speed of SPICE modeling
- The suggested improvements could be useful for most FETs compact models used in modern compact circuit simulators

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## USED NOTATIONS

The insulator capacitance per unit area:

$$C_{ox} = \frac{\epsilon}{d}$$

Characteristic voltage related to drift velocity saturation in high electric field:

$$V_L = \frac{v_S L}{\mu}$$

The gate oxide capacitance per unit area for MOS structure:

$$\beta = \mu C_{ox} \frac{W}{L}$$

Centered gate voltage ( $V_T$  is a threshold voltage):

$$V_{GT} = V_{GS} - V_T$$

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