Approaches for Analytical (Compact) Modeling of Tunneling Current in MOS Transistors

Alexander Kloes
Technische Hochschule Mittelhessen – University of Applied Sciences
NanoP – Competence Center for Nanotechnology and Photonics
Giessen, Germany
Motivation

Aggressive device scaling in new technologies: tunneling currents getting more important

Leakage effects to avoid:
- Gate leakage
- Source-to-drain tunneling (L << 10 nm)
- ...

New devices based on tunneling:
- Schottky barrier FET (SB-FET)
- Tunnel-FET (TFET)
- ...

Compact modeling must provide solutions to describe tunneling currents in a physics-based and numerically efficient way!
1. Basics of Tunneling
2. Approaches:
   - WKB Approximation
   - Wavelet transform
3. Results for Device Modeling:
   - Compact model for TFET \((WKB)\)
   - Ultra-Short Channel MOSFET \((WKB \text{ vs. Wavelet})\)
4. Conclusions
Outline

1. Basics of Tunneling

2. Approaches:
   - WKB Approximation
   - Wavelet transform

3. Results for Device Modeling:
   - Compact model for TFET (*WKB*)
   - Ultra-Short-Channel MOSFET (*WKB* vs. *Wavelet*)

4. Conclusions
1. Basics of Tunneling

- Solution of (at least) 1D time-independent Schrödinger equation necessary:

\[ -\frac{\hbar^2}{2m^*} \frac{d^2 \Psi(x)}{dx^2} + U(x) \cdot \Psi(x) = E \cdot \Psi(x) \]

- \( T_{\text{tun}} \): tunneling probability

- Analytical approximations possible?
Outline

1. Basics of Tunneling

2. Approaches:
   - WKB Approximation
   - Wavelet transform

3. Results for Device Modeling:
   - Compact model for TFET \((WKB)\)
   - Ultra-Short-Channel MOSFET \((WKB \text{ vs. Wavelet})\)

4. Conclusions
2. WKB Approach

**Wentzel-Kramers-Brillouin Approximation:**

- **Assumption:** $U(x)$ varies “slowly” with $x$:

  $$T_{\text{tun}} = \exp \left( -2 \cdot \int_{x_1}^{x_2} \sqrt{\frac{2m^* \cdot (U(x) - E)}{\hbar^2}} \, dx \right)$$

- **Closed form for triangular barrier:**

  $$T_{\text{tun}} = \exp \left( -\frac{4}{3} \sqrt{\frac{2m^*}{\hbar^2}} \cdot \frac{U(x_1) - E}{x_1 - x_2} \cdot \left( x_2^{3/2} - x_1^{3/2} \right) \right)$$

- **Kane’s tunneling model:**
  - based on WKB with const. field: $T \neq f(E)$
  - Integration w.r.t. energy replaced by effective DOS

2. WKB Approach

Wentzel-Kramers-Brillouin Approximation:

- Assumption: $U(x)$ varies “slowly” with $x$:

$$T_{\text{tun}} = \exp \left( -2 \cdot \int_{x_1}^{x_2} \sqrt{\frac{2m^* \cdot (U(x) - E)}{\hbar^2}} \, dx \right)$$

- Closed form for triangular barrier:

$$T_{\text{tun}} = \exp \left( -\frac{4}{3} \cdot \sqrt{\frac{2m^*}{\hbar^2}} \cdot \frac{U(x_1) - E}{x_1 - x_2} \cdot \left( x_2^{3/2} - x_1^{3/2} \right) \right)$$

- Kane’s tunneling model:
  - based on WKB with const. field: $T \neq f(E)$
  - Integration w.r.t. energy replaced by effective DOS

- Problem:
  at turning points $U(x)$ can be very steep!

2. Wavelet Approach

Wavelet Approach:

- Considering the Schrödinger equation:

\[
\frac{d^2 \psi}{dx^2} - k^2 \psi(x) = 0 \quad K = k^2 = \frac{2m^* \cdot (U(x) - E)}{\hbar^2}
\]

- Solution is approximated by **Shannon wavelet family**:

\[
\psi(x) = \alpha \varphi(x) + \overline{\alpha \varphi}(x) + \beta \eta(x) + \overline{\beta \eta}(x)
\]
2. Wavelet Approach

- By projection of Schrödinger equation an equivalent rectangular barrier is derived:

\[
K_{eq}(x) = \frac{1}{2\pi} \int_{-2\pi}^{2\pi} \hat{K}(\omega) d\omega
\]

- Equivalent barrier results in approx. same \(T_{\text{tun}}\) as \(U(x)\)
- Implementation requires FFT of potential barrier

1. Basics of Tunneling

2. Approaches:
   - WKB Approximation
   - Wavelet transform

3. Results for Device Modeling:
   - Compact model for TFET (*WKB*)
   - Ultra-Short-Channel MOSFET (*WKB vs. Wavelet*)

4. Conclusions
3. Compact Model for TFET

Tunnel-FET:

- Swing < 60 mV/dec possible
- Band-to-band tunneling at source junction
3. Compact Model for TFET

Area-equivalent WKB approximation:

- Improved representation of barrier
- Not relying on electric field in single point

![Diagram of TFET with energy levels and barrier heights](image)
3. Compact Model for TFET

DC/AC compact model in VerilogA:
- Verified by TCAD and meas. (FZ Juelich)
- Soon available on nanohub.org

VTC of inverter

Simulation of ring oscillator

Ultra-short channel MOSFET:

- Calculation of S/D tunneling
- For verification, potential extracted from TCAD
3. Ultra-SC MOSFET: WKB vs. Wavelet

- WKB approximation overestimates tunneling current
- Wavelet approach in good agreement to NEGF

Outline

1. Basics of Tunneling

2. Approaches:
   - WKB Approximation
   - Wavelet transform

3. Results for Device Modeling:
   - Compact model for TFET (WKB)
   - Ultra-Short-Channel MOSFET (WKB vs. Wavelet)

4. Conclusions
Conclusions

- Tunneling currents are getting more important in new devices
- Compact modeling must provide numerically efficient solutions

**WKB:**
- analytical solution for single energy level (triangular approximation)
- Kane’s model: effective DOS (instead of integration $dE$)
- can be improved by „area equivalent“ WKB
- fails if steep barrier profile

**Wavelet approach:**
- promising (better than WKB even for steep profile)
- numerically more expensive: FFT of barrier for each energy level
  ($\rightarrow$ analytical approximations?)
Acknowledgements

- German Federal Ministry of Education and Research under contract No. FKZ 03FH010IX5
- Forschungszentrum Jülich
- AdMOS – Advanced Modeling Solutions
- Universitat Rovira i Virgili, Tarrgona