An Enhanced Line-Reflect-Reflect-Match Calibration

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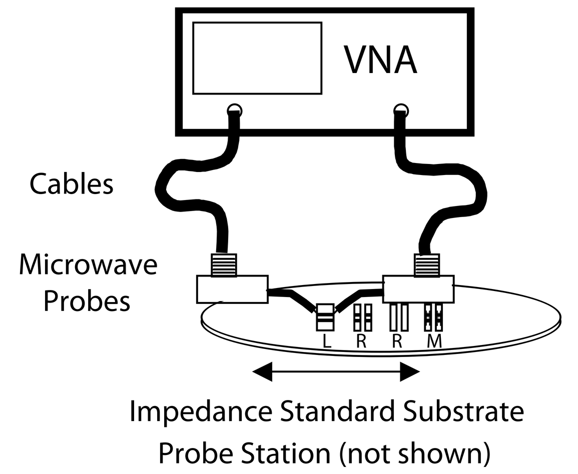
Motivation for this Work:

LRRM algorithm often works well…

- Probe placement tolerant
- Practical industry standard
- Compares favorably w/TRL
- Uses SOLT standard set



…but has limitations.

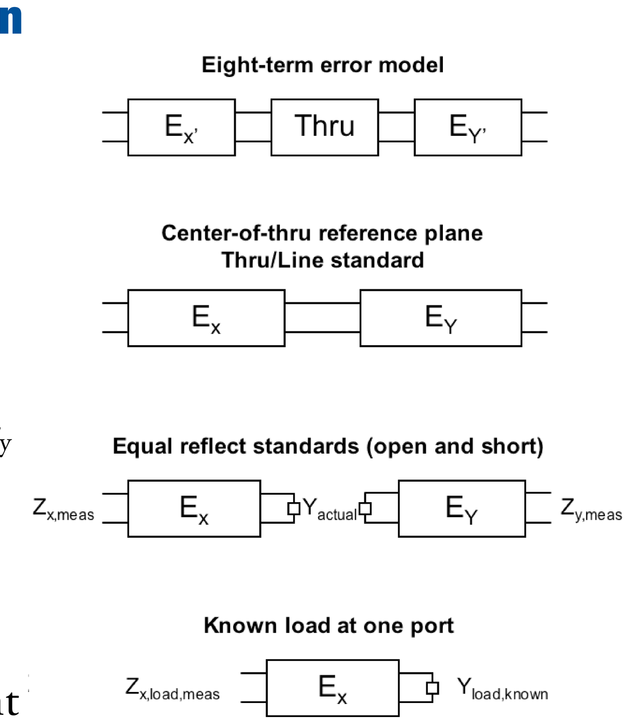
- Sensitive to long Thru delays
- Sensitive to high load inductance
- Inaccurate when $R_{load} \neq Z_{o,sys}$
- Off-wafer standard cal
- Does not account for cal std-DUT launch difference

Can we improve this?

Line-Reflect-Reflect-Match Calibration

Identify error boxes E_x and E_y
- Use ABCD parameters
- Line/Thru measurement
  - Relates E_y terms to E_x terms
- Equal port reflect standards
  - Two cases e.g., open & short
  - Additional equations for E_x & E_y
  - Solves A_x/C_x, B_x/D_x
- Known single load
  - Solves C_x/D_x
  - Enough to Cal
  - E_y from E_x and line measurement

All calculations at Thru-center plane



Automatic Load Inductance Extraction

Note that some ABCD terms are independent of load
- Perfect open: $Y_{X,actual} = 0$ $Y_{X,meas} = C_X/A_X$
- Perfect short: $Y_{X,actual} \rightarrow$ infinity, $Y_{X,meas} = D_X/B_X$

Only one term in correction equation depends on load
- C_X/D_X provides the correction
- $Y_{X,est} = C_X/D_X \cdot$ (load independent term)

Use lossless open reflect to solve for load reactance
- Estimate C_X/D_X, solve for $Y_{X,est}$, estimate to actual ratio of Y = est-act ratio of C/D
- Assume $Re(Y_{act}) = 0 \rightarrow L_{load}/R_{load}$ with R_{load} known

C_X/D_X known $\rightarrow$ E_X known $\rightarrow$ E_Y known $\rightarrow$ cal completed

LRRM Equations

The LRRM algorithm begins with solving for the error terms to the center of the Thru reference plane as shown in Fig.2(b). Once this process is completed then the known Thru behavior is used to move the reference plane to the probe tips. A careful observer will note that it is not just the Thru that must be known but actually the behavior of the two mirror-identical half-circuits that in cascade are equal to the Thru.

Expressing the cross-talk and switching term corrected measured Thru standard ABCD parameters, E_{MT}, as the cascade product gives:

$$E_{MT} = E_{X'} \cdot E_T \cdot E_{Y'} = E_{X'} \cdot E_{T/2} \cdot E_{T/2} \cdot E_{Y'} = E_X \cdot E_Y \quad (1)$$

where the $E_{T/2}$ terms represent the behavior of the half-thru structure and the probe tip reference plane error boxes $E_{X'}$ and $E_{Y'}$ can be found from the center-of-thru reference plane error boxes E_X and E_Y using:

$$E_{X'} = E_X \cdot (E_{T/2})^{-1} \quad (2)$$

and

$$E_{X'} = E_X \cdot (E_{T/2})^{-1}. \quad (3)$$

The normalized ABCD parameters of the error boxes are what we seek. Choosing D_X as the one term to leave unknown we have:

$$E_X = \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} = \begin{bmatrix} \underline{A}_X & \underline{B}_X \\ \underline{C}_X & \underline{D}_X \end{bmatrix} \cdot D_X \quad (4)$$

with

$$\begin{bmatrix} \underline{A}_X & \underline{B}_X \\ \underline{C}_X & \underline{D}_X \end{bmatrix} \equiv \begin{bmatrix} \frac{A_X}{D_X} & \frac{B_X}{D_X} \\ \frac{C_X}{D_X} & 1 \end{bmatrix}, \quad (5)$$

and

$$E_Y = \begin{bmatrix} A_Y & B_Y \\ C_Y & D_Y \end{bmatrix} = \begin{bmatrix} \underline{A}_Y & \underline{B}_Y \\ \underline{C}_Y & \underline{D}_Y \end{bmatrix} \cdot \frac{1}{D_X} \quad (6)$$

with

$$\begin{bmatrix} \underline{A}_Y & \underline{B}_Y \\ \underline{C}_Y & \underline{D}_Y \end{bmatrix} \equiv \begin{bmatrix} A_Y \cdot D_X & B_Y \cdot D_X \\ C_Y \cdot D_X & D_Y \cdot D_X \end{bmatrix}. \quad (7)$$

Since $E_{MT} = E_X \cdot E_Y$ we can determine E_Y from E_{MT} once we know E_X using:

$$E_Y = (E_X)^{-1} \cdot E_{MT} \quad (8)$$

$$E_Y = \frac{1}{D_X} \cdot \frac{1}{\underline{A}_X - \underline{B}_X \cdot \underline{C}_X} \cdot \begin{bmatrix} 1 & -\underline{B}_X \\ -\underline{C}_X & \underline{A}_X \end{bmatrix} \cdot \begin{bmatrix} t_1 & t_2 \\ t_3 & t_4 \end{bmatrix}. \quad (9)$$

where the E_{MT} matrix is known from the measurement term-by-term:

$$E_{MT} = \begin{bmatrix} t_1 & t_2 \\ t_3 & t_4 \end{bmatrix}. \quad (10)$$

The next tool we need is a set of general expressions that allow us to relate the measured behavior of a one-port termination with the actual behavior of the standard. We can get the measured impedance from actual admittance using:

$$Z_{x,meas} = \frac{\underline{A}_X + \underline{B}_X \cdot Y_{x,act}}{\underline{C}_X + Y_{x,act}}, \quad (11)$$

and

$$Z_{y,meas} = \frac{\underline{B}_Y \cdot Y_{y,act} + \underline{D}_Y}{\underline{A}_Y \cdot Y_{y,act} + \underline{C}_Y} \quad (12)$$

for ports X and Y respectively. The inverse expressions are used for correction and are:

$$Y_{x,act} = \underline{C}_X \cdot \frac{Z_{x,meas} - \frac{\underline{A}_X}{\underline{C}_X}}{\underline{B}_X - Z_{x,meas}}, \quad (13)$$

and

$$Y_{y,act} = \frac{\underline{C}_Y \cdot Z_{y,meas} - \underline{D}_Y}{\underline{B}_Y - \underline{A}_Y \cdot Z_{y,meas}}. \quad (14)$$

For the condition of a reflect pair standard providing equal actual admittance at both ports we can equate (13) with (14) and using (9) identify the expression:

$$P_1 \cdot a_1 + P_2 \cdot a_2 = V_a \quad (15)$$

where

$$P_1 = \left(\frac{\underline{A}_X}{\underline{C}_X} + \underline{B}_X\right), \quad P_2 = \left(\frac{\underline{A}_X \cdot \underline{B}_X}{\underline{C}_X}\right), \quad (16)\text{-}(17)$$

$$a_1 = t_1 \cdot Z_{y,meas} - t_2 + t_3 \cdot Z_{x,meas} \cdot Z_{y,meas} - t_4 \cdot Z_{x,meas}, \quad (18)$$

$$a_2 = 2 \cdot t_4 - 2 \cdot t_3 \cdot Z_{y,meas}, \quad (19)$$

and

$$V_a = 2 \cdot t_1 \cdot Z_{x,meas} \cdot Z_{y,meas} - 2 \cdot t_2 \cdot Z_{x,meas}. \quad (20)$$

For the second reflect standard we get a second expression similar to (15):

$$P_1 \cdot b_1 + P_2 \cdot b_2 = V_b \quad (21)$$

where the b_1, b_2, and V_b terms are found using (18)-(20) except using the measured impedances from the second pair of reflects.

The two equations (15) and (21) may be solved for the two unknowns yielding:

$$P_1 = \frac{V_a \cdot b_2 - V_b \cdot a_2}{a_1 \cdot b_2 - a_2 \cdot b_1} \quad (22)$$

and

$$P_2 = \frac{V_b \cdot a_1 - V_a \cdot b_1}{a_1 \cdot b_2 - a_2 \cdot b_1}. \quad (23)$$

From the definitions of P_1 and P_2 in (16) and (17) we form a quadratic equation with roots $\underline{A}_X/\underline{C}_X$ and $\underline{B}_X$

$$\underline{B}_X^{\,2} - P_1 \cdot \underline{B}_X + P_2 = 0 \quad (24)$$

or

$$\left(\frac{\underline{A}_X}{\underline{C}_X}\right)^2 - P_1 \cdot \left(\frac{\underline{A}_X}{\underline{C}_X}\right) + P_2 = 0, \quad (25)$$

with solutions given by:

$$\frac{\underline{A}_X}{\underline{C}_X}, \underline{B}_X = \frac{P_1 \pm \sqrt{P_1^2 - 4 \cdot P_2}}{2} \quad (26)$$

where the root selection is determined by trial and error using the needed sign of the corrected open reflection coefficient.

If we have a termination (e.g., load) with known behavior at the center-of-thru reference plane, we can determine $\underline{C}_X$ from a variation on (13):

$$\underline{C}_X = Y_{x,act,load} \cdot \frac{\underline{B}_X - Z_{x,meas,load}}{Z_{x,meas,load} - \frac{\underline{A}_X}{\underline{C}_X}} \quad (27)$$

or alternatively we can determine the $\underline{C}_X$ term using the automatic load inductance extraction process outlined below.

Once the $\underline{C}_X$ is known and applying (9) we have complete determination of the normalized error boxes at the center-of-thru reference plane. Using (2) and (3) the reference planes are moved to the probe tips. In normal application the probe tip error box ABCD parameters are converted to S-parameters and the eight-term error model is converted to a twelve-term model using switching terms and cross-talk terms identified when originally computing the eight-term error model reduction.

Load Inductance Equations

Using a variation of (11)

$$Y_{x,meas} = \frac{\underline{C}_X + Y_{x,act}}{\underline{A}_X + \underline{B}_X \cdot Y_{x,act}} \quad (28)$$

we note the following cases:

i. Perfect open, $Y_{X,act} = 0$, $Y_{X,meas} = \underline{C}_X/\underline{A}_X$

ii. Perfect short, $Y_{X,act} \rightarrow$ infinity, $Y_{X,meas} \rightarrow 1/\underline{B}_X$.

These terms are independent of the load definition used in (27) and solely determined by the open and short.

If we make an estimate of $\underline{C}_X$ and use it to complete the correction then the resultant estimate correction of a measurement at port X would be given by:

$$Y_{x,est} = \underline{C}_{X,est} \cdot \frac{Z_{x,meas} - \frac{\underline{A}_X}{\underline{C}_X}}{\underline{B}_X - Z_{x,meas}}. \quad (29)$$

Forming the ratio of (29) for the two situations where an estimate is used and where the actual $\underline{C}_X$ is used results in a simple relation since the fractional part of (29) drops out:

$$\frac{Y_{x,est}}{Y_{x,act}} = \frac{\underline{C}_{X,est} \cdot \frac{Z_{x,meas} - \frac{\underline{A}_X}{\underline{C}_X}}{\underline{B}_X - Z_{x,meas}}}{\underline{C}_{X,act} \cdot \frac{Z_{x,meas} - \frac{\underline{A}_X}{\underline{C}_X}}{\underline{B}_X - Z_{x,meas}}} = \frac{\underline{C}_{X,est}}{\underline{C}_{X,act}} \equiv \alpha \quad (30)$$

Using the load extraction method described in [1]-[2] we assume an ideal load ($Y_{X,est,load} = 1 + j0$) in (29) to obtain the estimate $\underline{C}_{X,est}$. The ratio defined in (30) is determined solely by the ratio of the estimated load to the ideal load which will be the error ratio for measurement of any DUT:

$$Y_{x,est,dut} = \alpha \cdot Y_{x,act,dut} = \frac{Y_{x,est,load}}{Y_{x,act,load}} \cdot Y_{x,act,dut}. \quad (31)$$

For a reflect (e.g., open) standard known to be reactive only at the center-of-thru reference plane ($Y_{X,act,open} = 0 + jB_{open,act}$) the estimated behavior ($Y_{X,est,open} = G_{open,est} + jB_{open,est}$) is given by:

$$Y_{x,est,open} = \alpha \cdot Y_{x,act,open} = \alpha \cdot (0 + jB_{open,act}). \quad (32)$$

Remembering that the ratio term may be complex and equating the real parts of (32) means that:

$$real\left(\frac{1}{\alpha}\right) \cdot G_{open,est} - imag\left(\frac{1}{\alpha}\right) \cdot B_{open,est} = 0 \quad (33)$$

$$\Rightarrow \frac{imag\left(\frac{1}{\alpha}\right)}{real\left(\frac{1}{\alpha}\right)} = \frac{-\omega \cdot L_{act}}{R_{act}} = \frac{G_{open,est}}{B_{open,est}} \quad (34)$$

since

$$\frac{1}{\alpha} = \frac{Z_{x,est,load}}{Z_{x,act,load}} = \frac{Z_{x,est,load}}{Z_{x,act,load}} \cdot \frac{Z^*_{x,act,load}}{Z^*_{x,act,load}} \quad (35)$$

$$\Rightarrow \frac{1}{\alpha} = \frac{R_o}{|Z_{x,act,load}|^2} \cdot (R_{act} - j\omega \cdot L_{act}). \quad (36)$$

Solving (36) for the load inductance yields:

$$L_{act} = -\frac{R_{act} \cdot G_{open,est}}{\omega \cdot B_{open,est}}, \quad (37)$$

where R, G, and B are the real part of the actual load impedance and the components of the estimated behavior of the open standard using the perfect assumption, all at the center-of-thru reference plane.

The use of ABCD parameters, impedances, and admittances in the derivation avoids the possible problem associated with the implicit assumption of the existence of an intermediate reference impedance suggest in [14].

LRRM Assumptions and Limitations

Potential singularity with long Thru and probe-tip reflects
- Thru-center Γ_{refl} phase rotated from short/load
- Problems occur when $real(\Gamma_{refl}) = 0$, half-wavelength thru
- Workaround: Use alternative LRM w/auto load inductance

Thru impedance matched to the system impedance
- Insignificant error for electrically short Thru
- Problems occur for physically long Thru and high frequency

Thru/Line per-unit length behavior fully known
- Needed to set calibration reference plane to probe-tip
- Insignificant error for electrically short Thru
- Problems occur for long/lossy Thru

LOAD L Assumptions and Limitations

Load Z equal at probe-tip and Thru-center
- Insignificant error for electrically short Thru & small load L
- Problems occur for long Thru, high load L case
- Workaround: use offset reflects (near to Thru-center)

Zero open reflect loss at Thru-center
- Reflect is lossless at probe-tip, has half-Thru gain at center
- Small error for electrically short or lossless Thru
- Workaround: use offset reflects (near to Thru-center)

Primitive L correction when load R ≠ $Z_{o,sys}$
- Only useful for electrically very short Thru

Enhanced LRRM Methods

Eliminate assumption-heavy direct calculation method

Use multiple guesses of inductance values at probe-tip
- 50 points dense near initial center value but with wide range

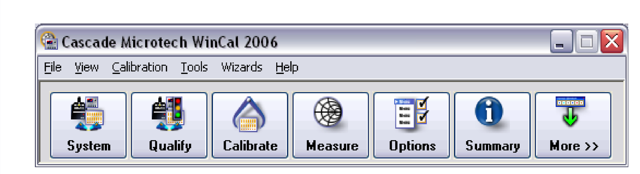
Translate load impedance guesses to Thru-center plane
- Perform reference plane translations using complete line model including loss and Zo

Compute error terms and open verification for each guess

Calculate expected open behavior for each guess

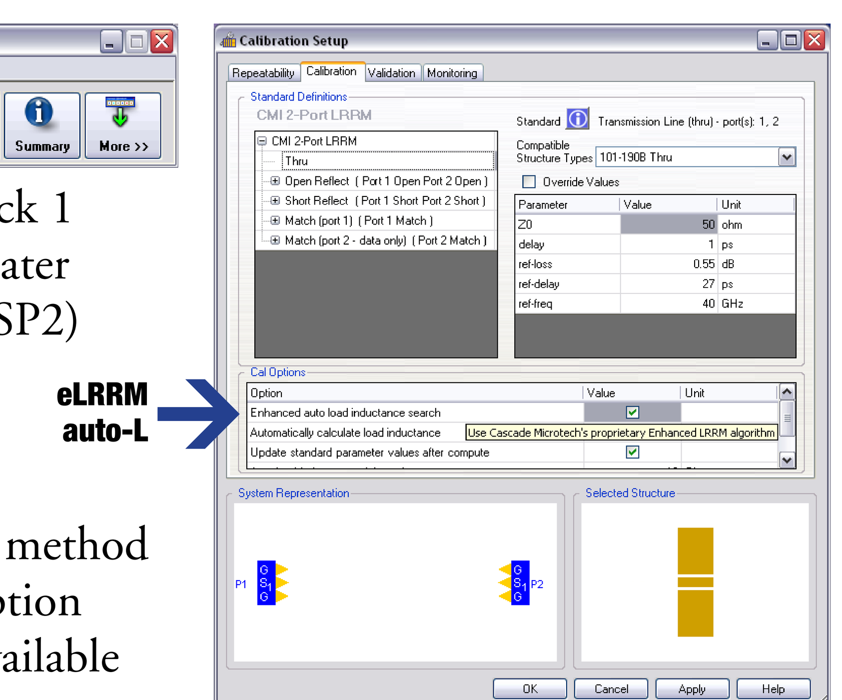
Select inductance guess that results in closest open reflection magnitude

eLRRM in WinCal 2006



Available in service pack 1 (version 4.01) and later

Current release 4.02 (SP2)



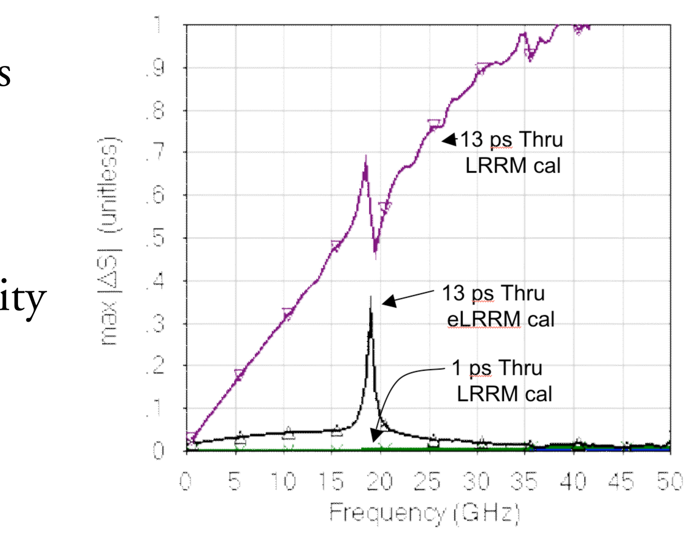
eLRRM
- Load inductance method enabled by cal option
- Other features available when standard parameters warrant

eLRRM Cal Comparison Results

Worst-case cal comparisons
- First tier NIST method
- Reference is 1 ps thru eLRRM (load L = 3.7 pH)

13 ps Thru, probe-tip reflects
- Non-independent reflects has singularity at 19 GHz
- LRRM fails miserably
  - Load L confused (155 pH)
- eLRRM is robust to singularity
  - Successful L (2.8 pH)

1 ps Thru LRRM (no singularity)
- Successful L (2.6 pH)

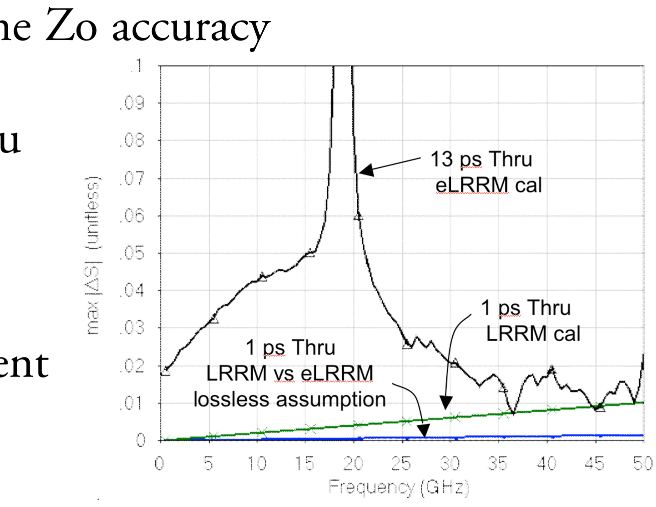


13 ps Thru eLRRM
- Error near reflects singularity
- Other error only limited by line Zo accuracy

1 ps eLRRM is comparison ref
- Accounts for small loss of Thru

1 ps Thru LRRM
- Assumes no loss
- Shows only small deviation

Lossless eLRRM & LRRM equivalent
- Essentially identical for common case
- LRRM validated to NIST MultiCal



10x Expanded Y-scale

Cal Comparison Significance

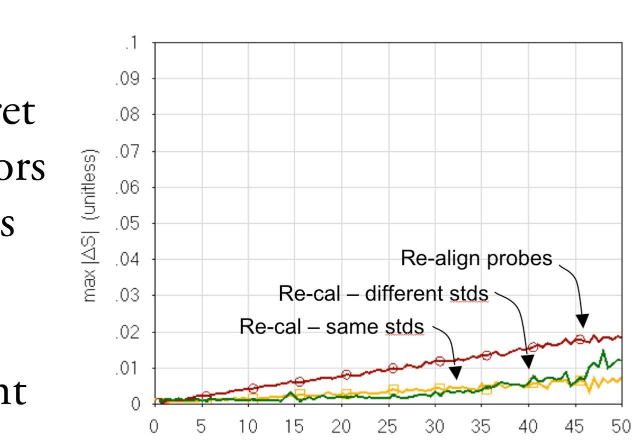
Error magnitude significance
- Use these curves to help interpret the scale and importance of errors

Simple repeatability – same standards
- Repeat an auto-cal sequence

Standard variation
- Repeat an auto-cal with different standard set

Probe positioning (dominant error)
- Randomize probes then carefully realign and repeat auto-cal

1 ps cal differences on prior page small comparable to repeatability limits



10x Expanded Y-scale

Summary (Pros & Cons)

LRRM is an excellent calibration method but like every cal method has limitations when assumptions don't hold

eLRRM provides a more robust calibration for:
- Electrically long Thru lines
- Thru line impedance deviating from system and load resistance
- Reflect standards not located at the Thru-center plane
- High load inductance

But…
- eLRRM may require more known behavior about structures to avoid making the assumptions in LRRM