Nonlinear Device Modeling with Scalable X-parameters*

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based on paper at IMS 2012

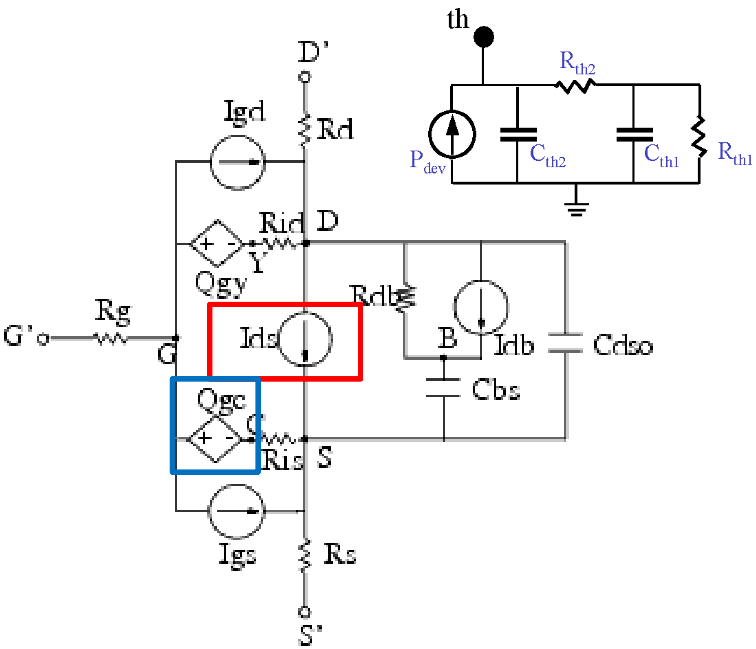
* “X-parameters” is a trademark of Agilent Technologies

Outline

- Introduction:

  compact and behavioral models
- X-parameters for transistors
- Scaling relations: derivations & results
- Validation: simulation and experiment
- Summary

Compact models



Coupled nonlinear equivalent circuits
(e.g. Electrical, thermal, trap capture / emission)

Each element has *constitutive relation* (I-V, Q-V)
Each constitutive relation has *many parameters*

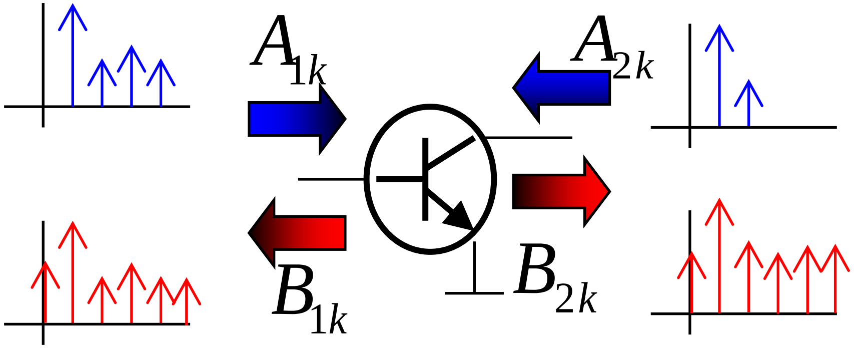
$$I(t) = Ids(V_1(t), V_2(t)) \qquad I(t) = \frac{d}{dt} Q_{gc}(V_1(t), V_2(t))$$

- Model has an assumed structure, ckt elements and topology
  - lumped approximation assumed valid
- Try to model all relevant physics effecting device behavior
  - invariably some physics is left out
- Work in all simulation modes (transient, harmonic balance, …)

Behavioral models

X-parameter Model

$$B_{p,k} = F_{p,k}(A_{1,1}, A_{1,2}, \ldots A_{2,1}, \ldots A_{2,N})$$



- model observable phenomena (input-output) effecting device behavior
- must be constructed from measurements (stimulus-response data) at external terminals of the device; no internal structure assumed
- not restricted to transistors
- limited “extrapolation” capabilities beyond actual data measured
- X-parameter behavioral models cannot simulate transient effects; dynamics can be obtained in the envelope domain

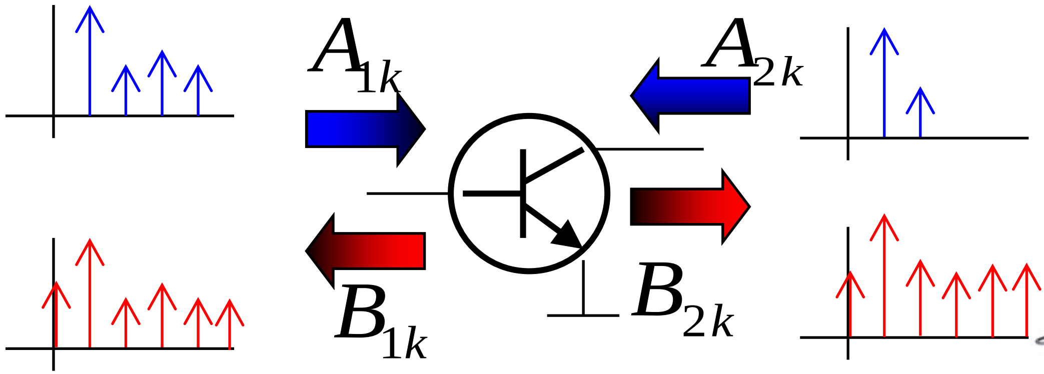
X-parameter philosophy

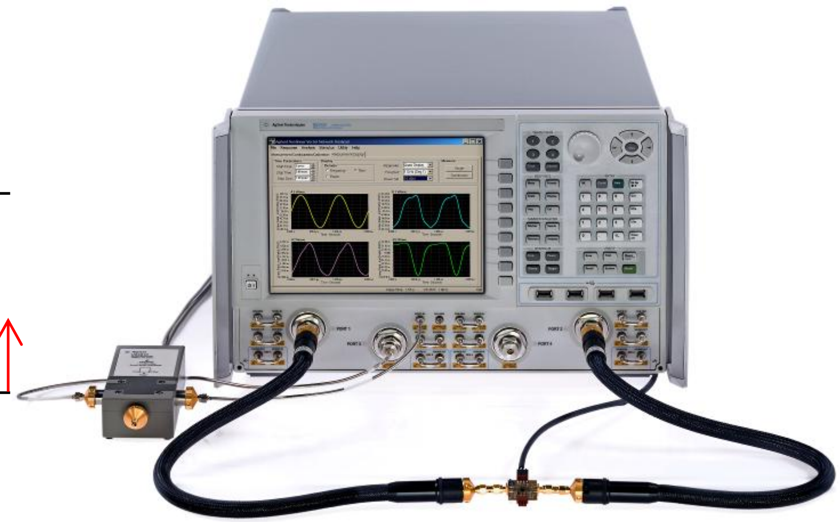
- X-parameters: based on algebraic complex frequency maps that are directly solved by Harmonic Balance simulators like Agilent ADS

- Why X-parameters?
  - Behavioral representation protects technology and model IP
  - Model well distributed circuits
  - X-parameters often converge better in nonlinear analysis than compact models.
  - Useful when good compact models are not available.
  - Works for entire circuits, not just transistors

X-parameters provide a complementary approach to transistor modeling

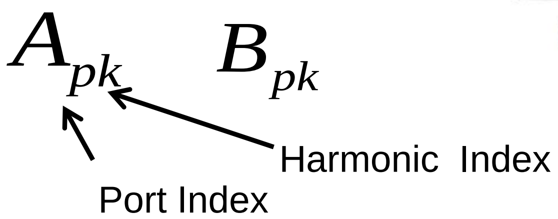
Wave variables, complex spectra and waveforms





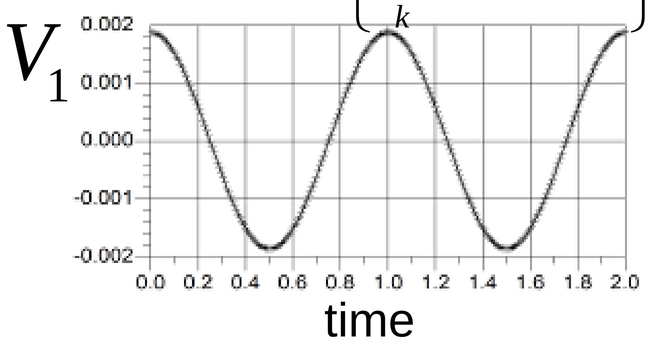
$$V_{p,k} = \sqrt{2Z_0} \cdot (A_{p,k} + B_{p,k})$$

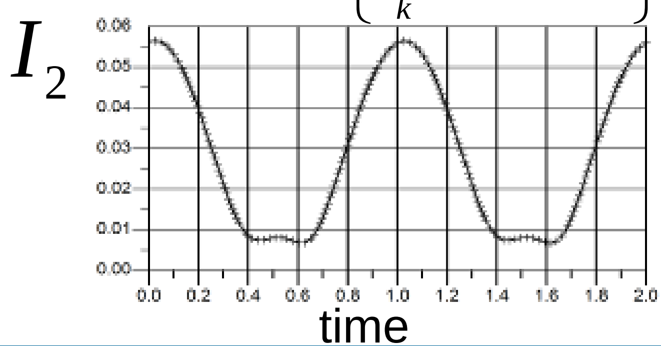
$$I_{p,k} = \sqrt{\frac{2}{Z_0}} \cdot \left(A_{p,k} - B_{p,k}\right)$$



$$v_p(t) = \mathrm{Re}\left\{\sum_k V_{p,k} e^{jk\omega_0 t}\right\}$$

$$i_p(t) = \mathrm{Re}\left\{\sum_k I_{p,k} e^{jk\omega_0 t}\right\}$$





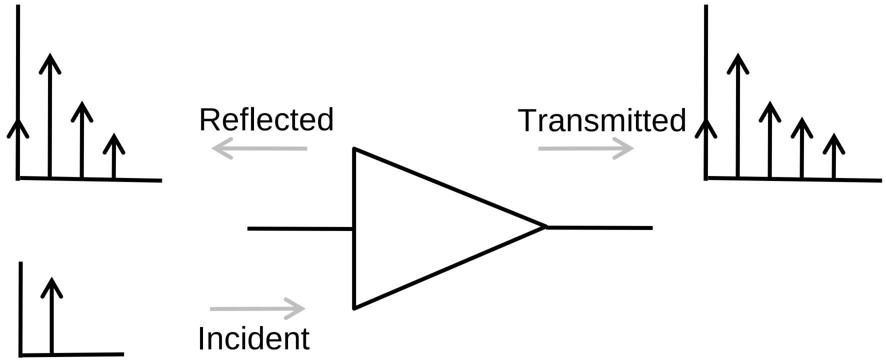
Superset of DC & S-parmeter data

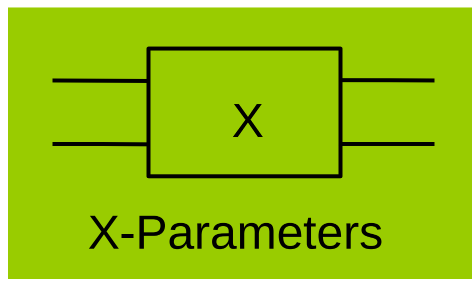
- S-parameters Waveforms
- Intermodulation
- X-parameters

X-parameters: same use model as linear S-parameters *but much more powerful*

Measure

Agilent Nonlinear Vector Network Analyzer



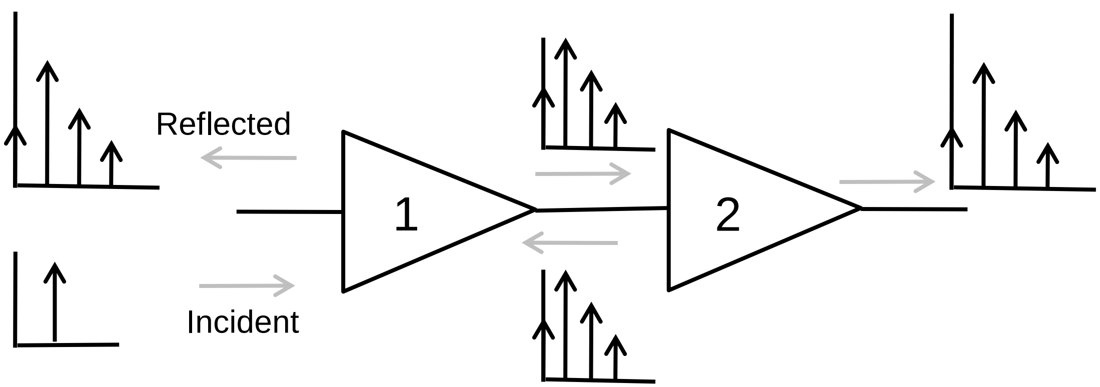


Model

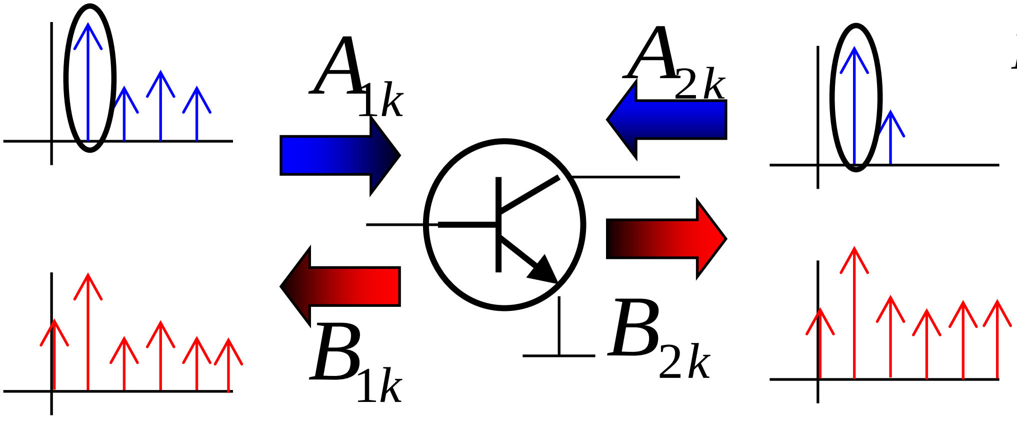
$$B = X(A)$$

Design

EDA Software



X-parameters for transistors



$$B_{2,k} = F_{2,k}(DC, A_{11}, A_{12}, \ldots, A_{21}, A_{22}, \ldots)$$

Harmonic index

Port index

$$F_{2,k}(DC, A_{11}, A_{12}, \ldots, A_{21}, \ldots, A_{22}, \ldots) =$$
$$F_{2,k}(DC, A_{11}P^{-1}, A_{21}P^{-1}, \ldots, A_{2,M}P^{-M})P^{k}$$
$$P = e^{j\phi(A_{1,1})}$$

Time invariance (requirement)

Spectral linearization (for convenience) around $LSOP = \left[V_1^{DC}, V_2^{DC}, |A_{1,1}|, |A_{2,1}|, \theta\right]$

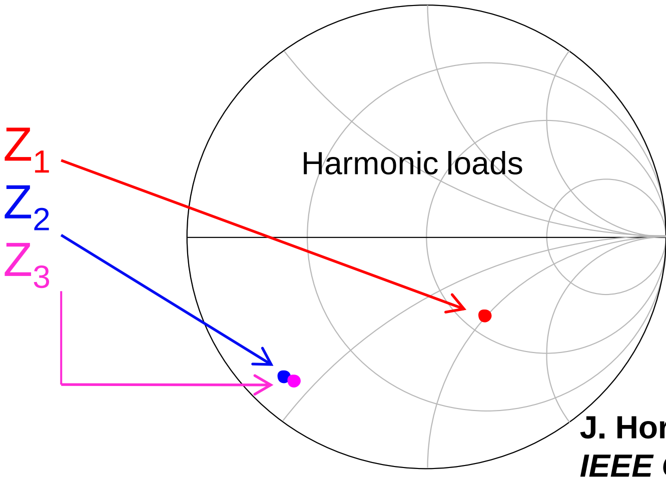
$$B_{p,k} \cong X^{(F)}_{p,k}(LSOP)P^{k} + \sum_{(q,m)>(1,1)} X^{(S)}_{p,k;q,m}(LSOP)P^{k-m}A_{q,m} + \sum_{(q,m)>(1,1)} X^{(T)}_{p,k;q,m}(LSOP)P^{k+m}A^{*}_{q,m}$$

$$I_p \cong X_p^{(FI)}(LSOP) + \sum \mathrm{Re}\left(X^{(Y)}_{p;q,l}(LSOP) \cdot A_{q,l}\right)$$

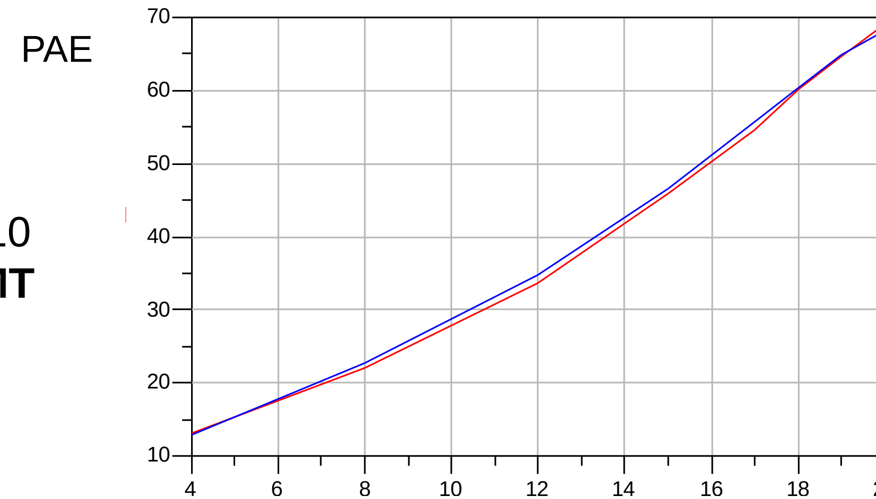
Outputs assuming all harmonics are matched

Cross-frequency mismatch sensitivity terms

GaN HEMT X-Parameter model



Cree
CGH40010
GaN HEMT



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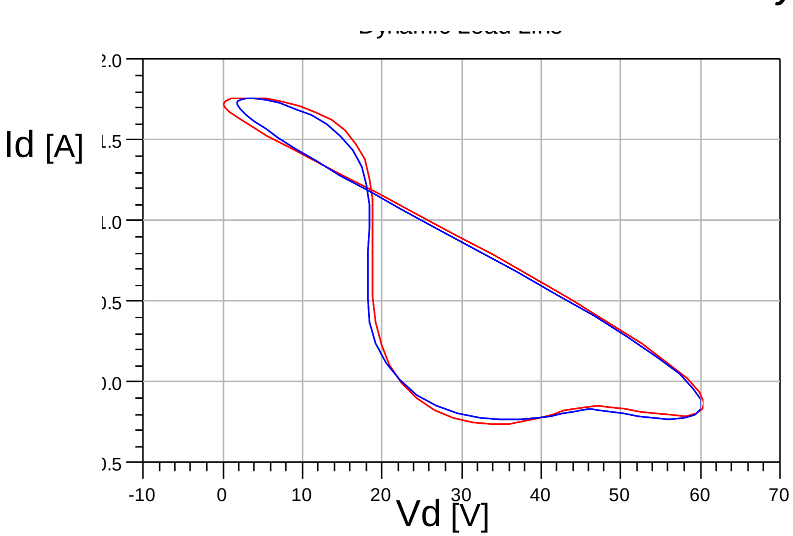
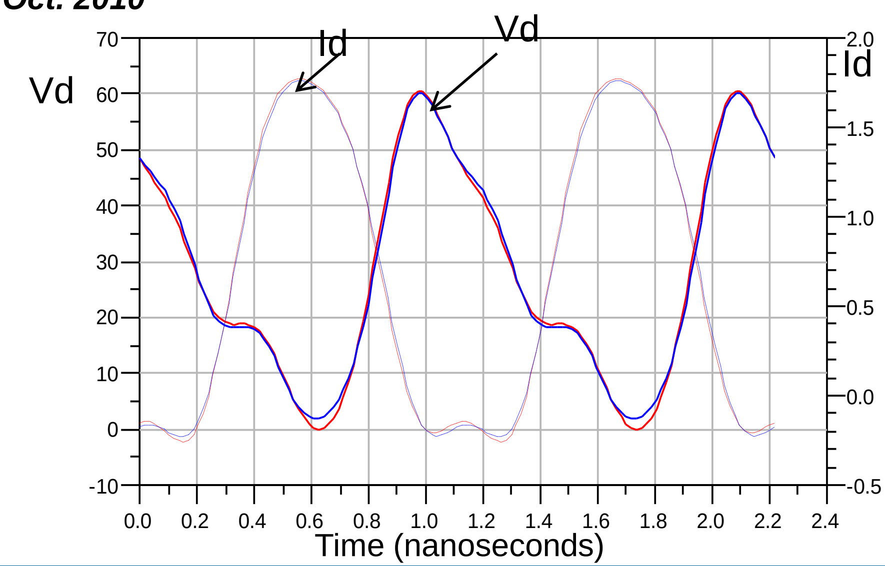
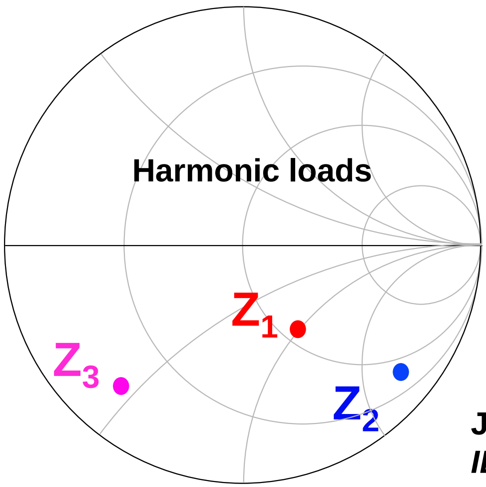




GaN HEMT X-Parameter model



Cree CGH40010
GaN HEMT

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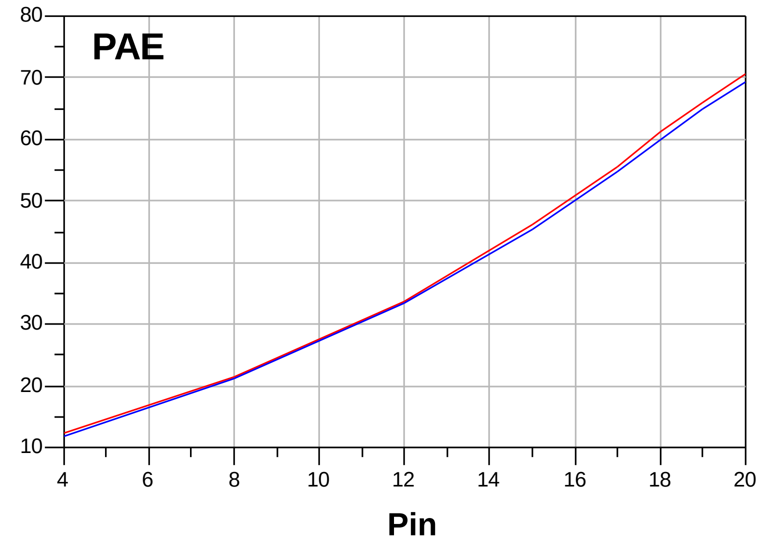
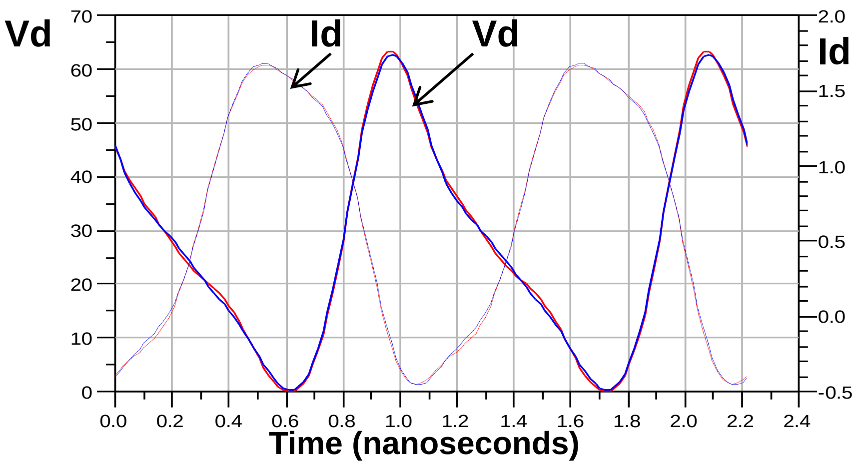
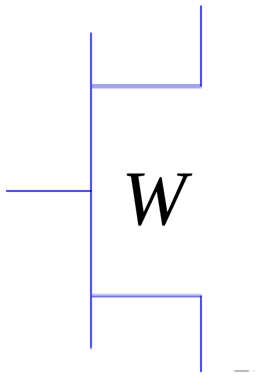




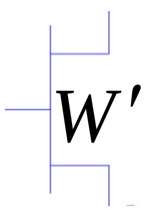
X-parameter model with fundamental load control
NVNA + *harmonic load-pull* measurements

want this accuracy in
a scalable model

Statement of problem



X-parameters of *DUT* are obtained



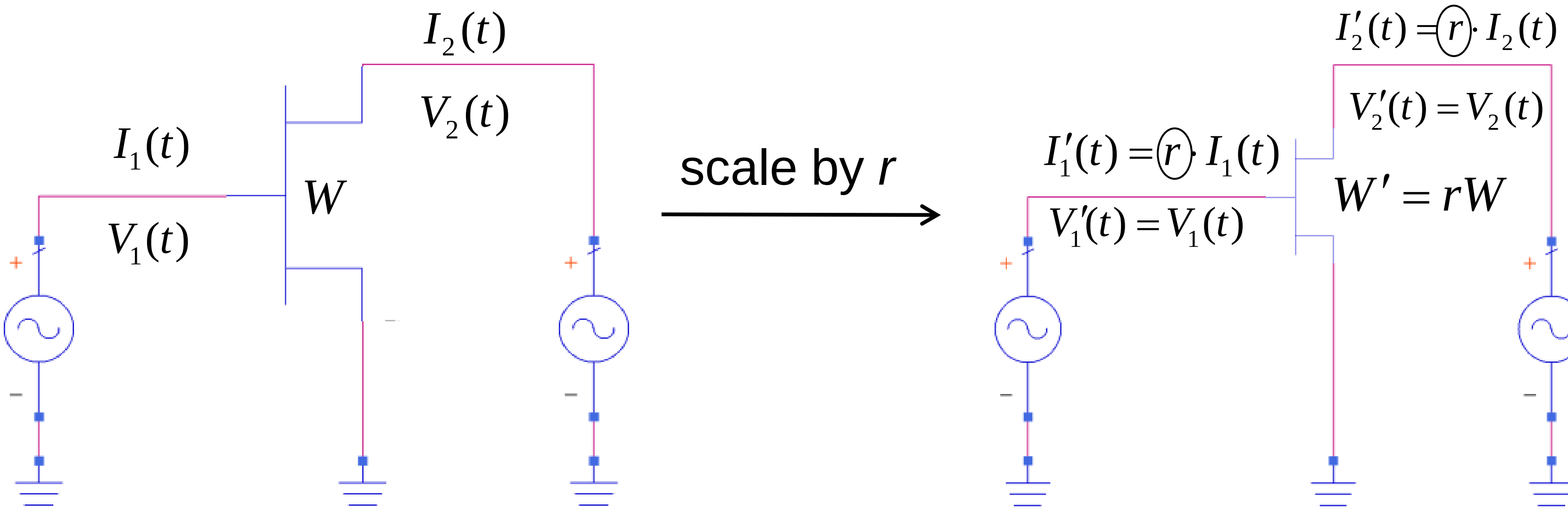
DUT’ has specified relationship to *DUT*

Predict the X-parameters for *DUT’* from those of *DUT*

$$B' = X'(A') \quad \Leftarrow \quad B = X(A)$$

Derivation 1: scaled ref. impedance

Simple (intrinsic) device scaling in time-domain



Convert to freq. domain

Define wave variables

$$A'_{p,k} = \frac{V'_{p,k} + Z'_0 I'_{p,k}}{2\sqrt{Z'_0}} = \frac{V_{p,k} + \left(\frac{Z_0}{r}\right) r I_{p,k}}{2\sqrt{\frac{Z_0}{r}}}$$

$$B'_{p,k} = \frac{V'_{p,k} - Z'_0 I'_{p,k}}{2\sqrt{Z'_0}} = \frac{V_{p,k} - \left(\frac{Z_0}{r}\right) r I_{p,k}}{2\sqrt{\frac{Z_0}{r}}}$$

apply scaling rules

convert *I & V* *back to* wave variables

$$V_{p,k} = \left(A_{p,k} + B_{p,k}\right)\sqrt{Z_0}$$

$$I_{p,k} = \frac{\left(A_{p,k} - B_{p,k}\right)}{\sqrt{Z_0}}$$

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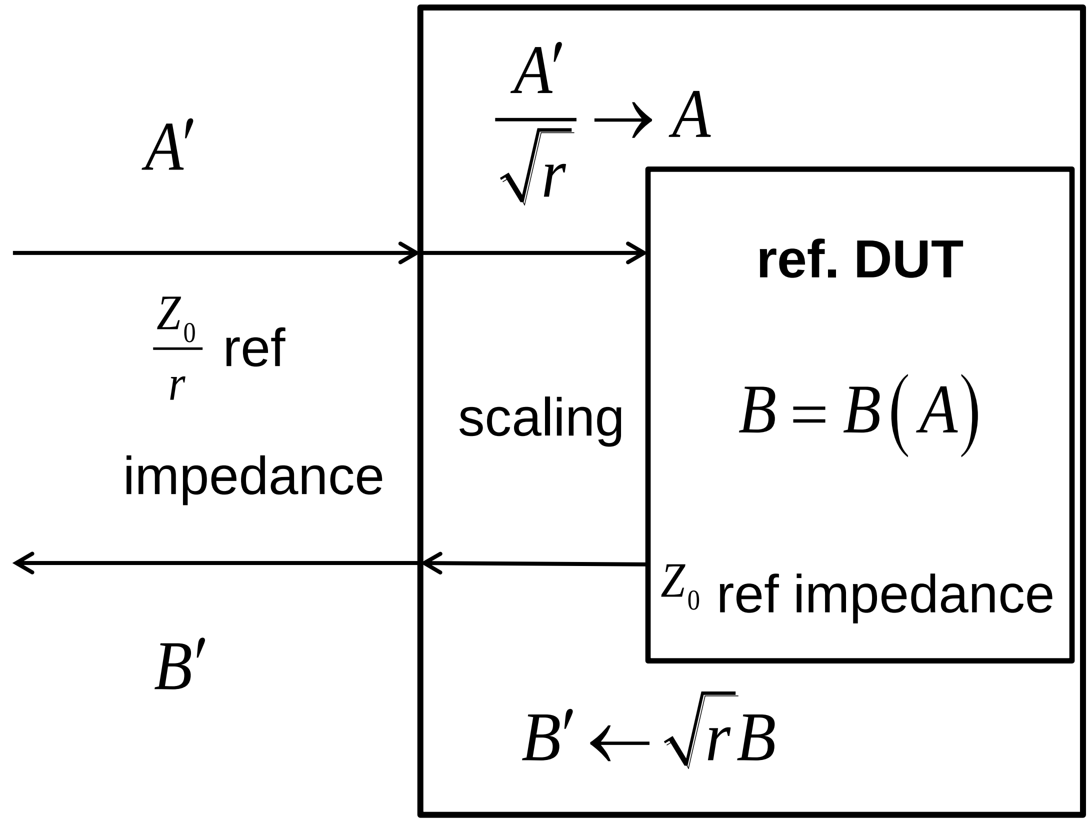
Results and Interpretation

$$A'_{p,k} = \sqrt{r}\,A_{p,k}$$

$$B'_{p,k} = \sqrt{r}\,B_{p,k}$$

independent scaling

$$B'(A') = \sqrt{r}\,B\left(\sqrt{r}\,A\right)$$



X-parameter Formulae (1)

$$A'_{p,k} = \sqrt{r} A_{p,k} \qquad B'_{p,k} = \sqrt{r} B_{p,k}$$

$$B'_{p,k} \approx X'^{(F)}_{p,k}(LSOP') P'^{k}$$

$$+ \sum_{(q,m)>(1,1)} X'^{(S)}_{p,k;q,m}(LSOP') P'^{k-m} A'_{q,m} + \sum_{(q,m)>(1,1)} X'^{(T)}_{p,k;q,m}(LSOP') P'^{k+m} A'^{*}_{q,m}$$

LSOP' LSOP

$$X'^{(F)}_{\frac{Z_0}{r}}\left(\sqrt{r}\,|A_{11}|, \sqrt{r} A_{21}\right) = \sqrt{r} \cdot X^{(F)}_{Z_0}\left(|A_{11}|, A_{21}\right)$$

X-par scale factor

$$X'^{(S)}_{\frac{Z_0}{r}}\left(\sqrt{r}\,|A_{11}|, \sqrt{r} A_{21}\right) = X^{(S)}_{Z_0}\left(|A_{11}|, A_{21}\right)$$

$$X'^{(T)}_{\frac{Z_0}{r}}\left(\sqrt{r}\,|A_{11}|, \sqrt{r} A_{21}\right) = X^{(T)}_{Z_0}\left(|A_{11}|, A_{21}\right)$$

X-parameter Formulae (2)

$$I'_p = rI_p \qquad V'_p = V_p$$

For voltage bias

$$I'_p \cong X'^{(FI)}_p(LSOP') + \sum \mathrm{Re}\left(X'^{(Y)}_{p;q,l}(LSOP') \cdot A'_{q,l}\right)$$

$$X'^{(FI)}_{\frac{Z_0}{r}}\left(\sqrt{r}|A_{11}|, \sqrt{r}A_{21}\right) = r \cdot X^{(FI)}_{Z_0}\left(|A_{11}|, A_{21}\right)$$

X-par scale factor

$$X'^{(Y)}_{\frac{Z_0}{r}}\left(\sqrt{r}|A_{11}|, \sqrt{r}A_{21}\right) = \sqrt{r} \cdot X^{(Y)}_{Z_0}\left(|A_{11}|, A_{21}\right)$$

For current bias

$$V'_p \cong X'^{(FV)}_p(LSOP') + \sum \mathrm{Re}\left(X'^{(Z)}_{p;q,l}(LSOP') \cdot A'_{q,l}\right)$$

$$X'^{(FV)}_{\frac{Z_0}{r}}\left(\sqrt{r}|A_{11}|, \sqrt{r}A_{21}\right) = X^{(FV)}_{Z_0}\left(|A_{11}|, A_{21}\right)$$

$$X'^{(Z)}_{\frac{Z_0}{r}}\left(\sqrt{r}|A_{11}|, \sqrt{r}A_{21}\right) = \frac{1}{\sqrt{r}} \cdot X^{(Z)}_{Z_0}\left(|A_{11}|, A_{21}\right)$$

Validation: Numerical (in ADS)

Procedure

Scalable intrinsic compact transistor model size W

Extract X-parameters using X-parameter Generator

Scale intrinsic compact model *to size W'=rW*

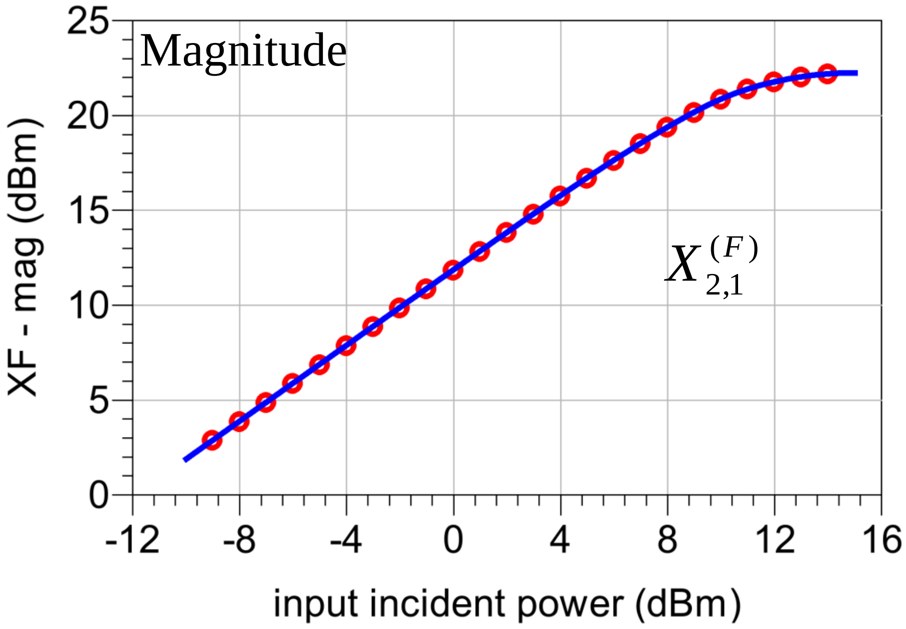
Scale X-parameters to *W'=rW* using explicit formulae

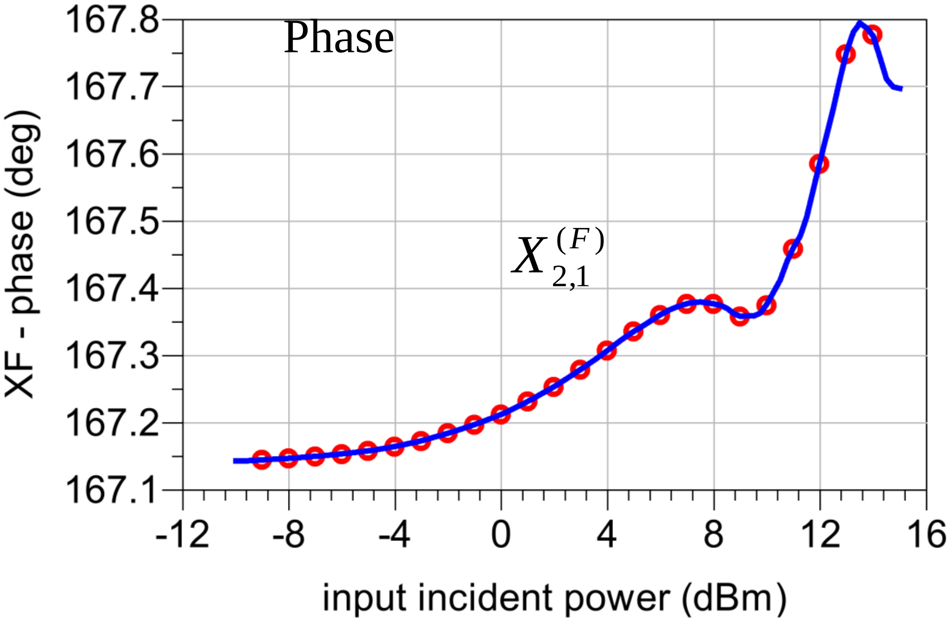
compare

Extract X-parameters using X-parameter generator in scaled Z_0

Validation: Numerical

Magnitude



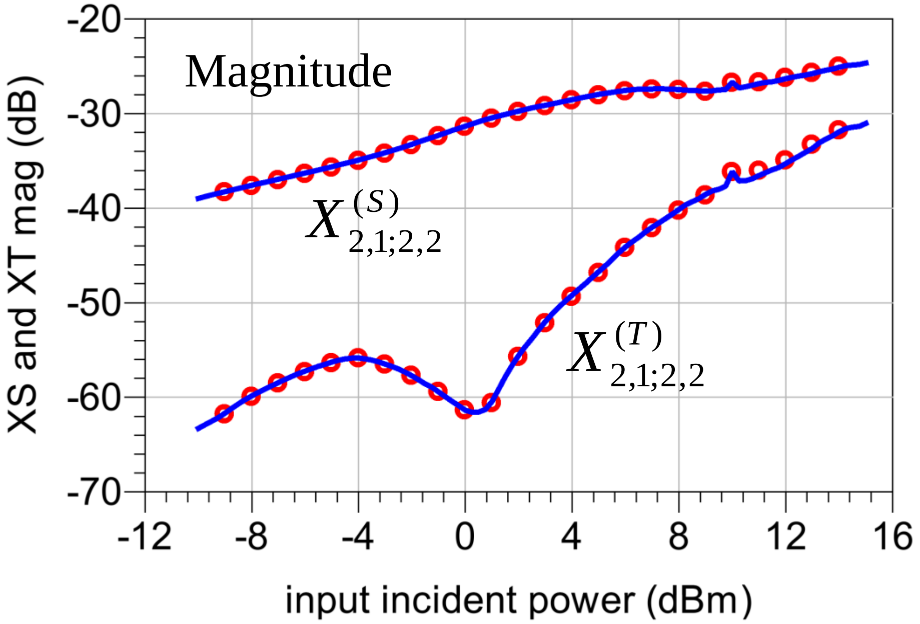


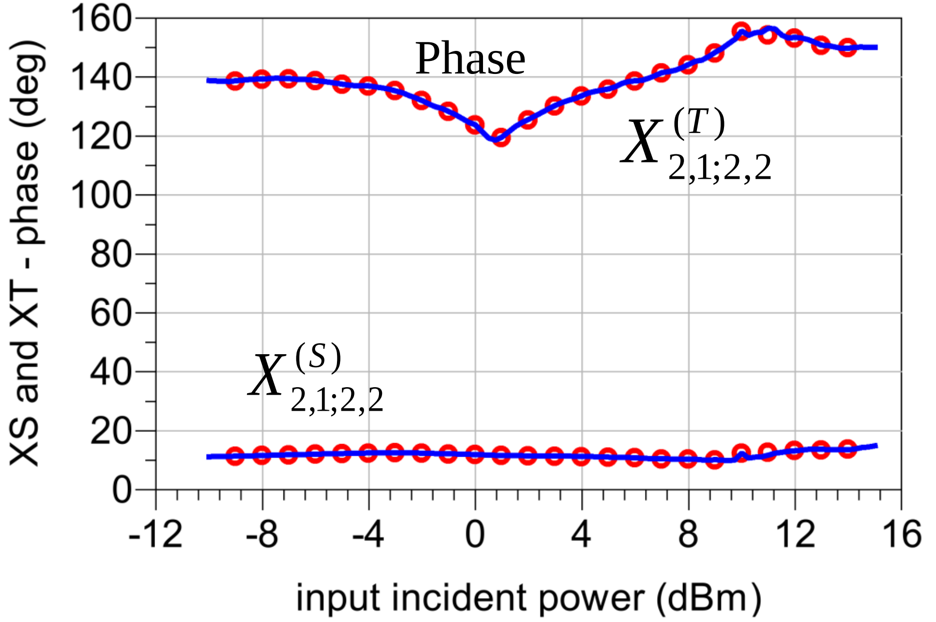
X-parameters of scaled compact model (red symbols)

Scaled X-parameters with scaled Z0 (blue line)

Validation: Numerical

Magnitude

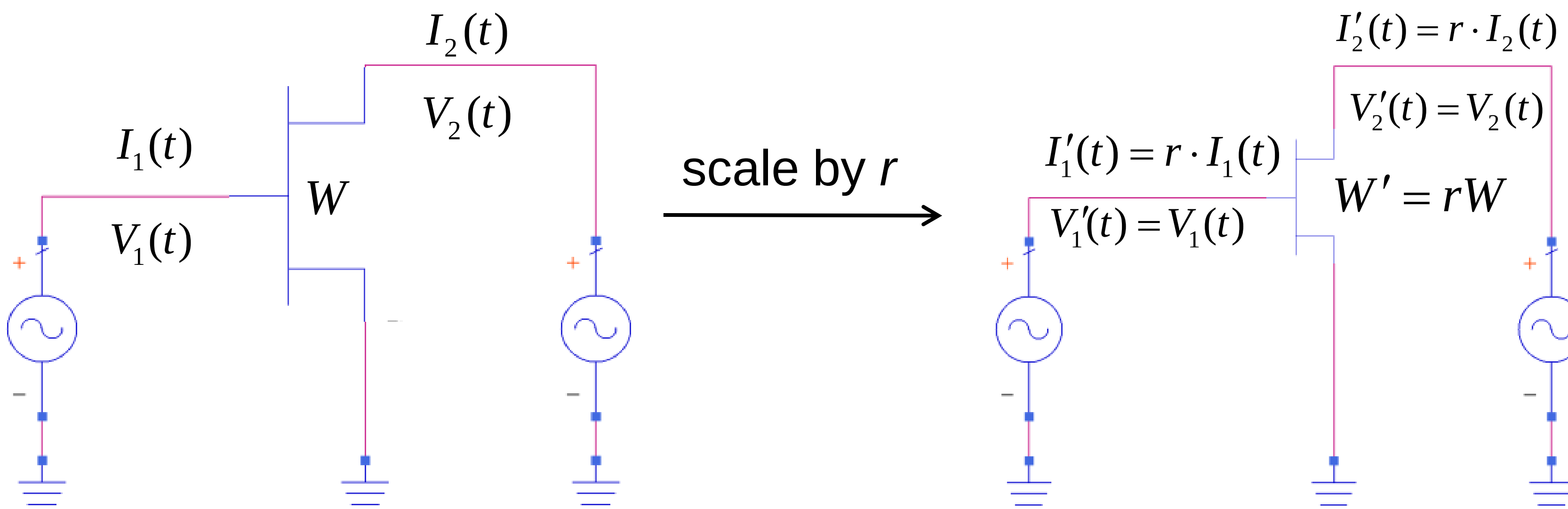




Scaled X-parameters with scaled Z0 (blue line)

X-parameters of scaled compact model (red symbols)

Derivation 2: fixed ref. impedance



$$A'_{p,k} = \frac{V'_{p,k} + Z'_0 I'_{p,k}}{2\sqrt{Z_0}} = \frac{V_{p,k} + Z_0 r I_{p,k}}{2\sqrt{Z_0}} = \left(\frac{1+r}{2}\right) A_{p,k} + \left(\frac{1-r}{2}\right) B_{p,k}$$

$$B'_{p,k} = \frac{V'_{p,k} - Z'_0 I'_{p,k}}{2\sqrt{Z'_0}} = \frac{V_{p,k} - Z_0 r I_{p,k}}{2\sqrt{Z_0}} = \left(\frac{1-r}{2}\right) A_{p,k} + \left(\frac{1+r}{2}\right) B_{p,k}$$

same Z_0

Implicit scaling

$$A' = \left(\frac{1+r}{2}\right) A + \left(\frac{1-r}{2}\right) X(A)$$

$$B' = \left(\frac{1-r}{2}\right) A + \left(\frac{1+r}{2}\right) X(A)$$

A' →

$$A' = \left(\frac{1+r}{2}\right) \bar{A} + \left(\frac{1-r}{2}\right) X(\bar{A})$$

solve for $\bar{A}$

evaluate

$$B' = \left(\frac{1-r}{2}\right) \bar{A} + \left(\frac{1+r}{2}\right) X(\bar{A})$$

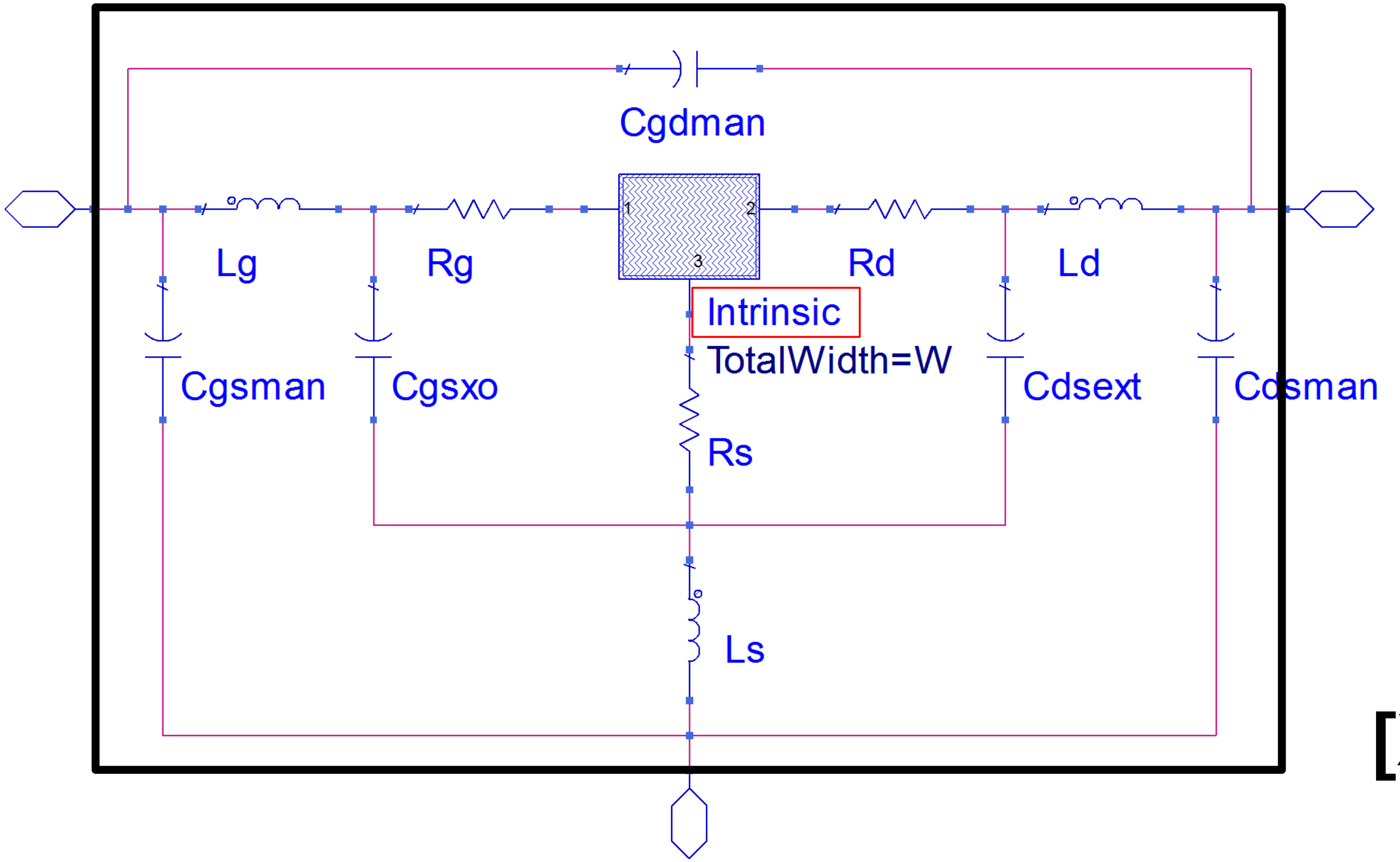
← B'

requires solution of nonlinear equations

Z_0 ref impedance

Application: Non-ideal Scaling

simple scaling only valid for *intrinsic* device



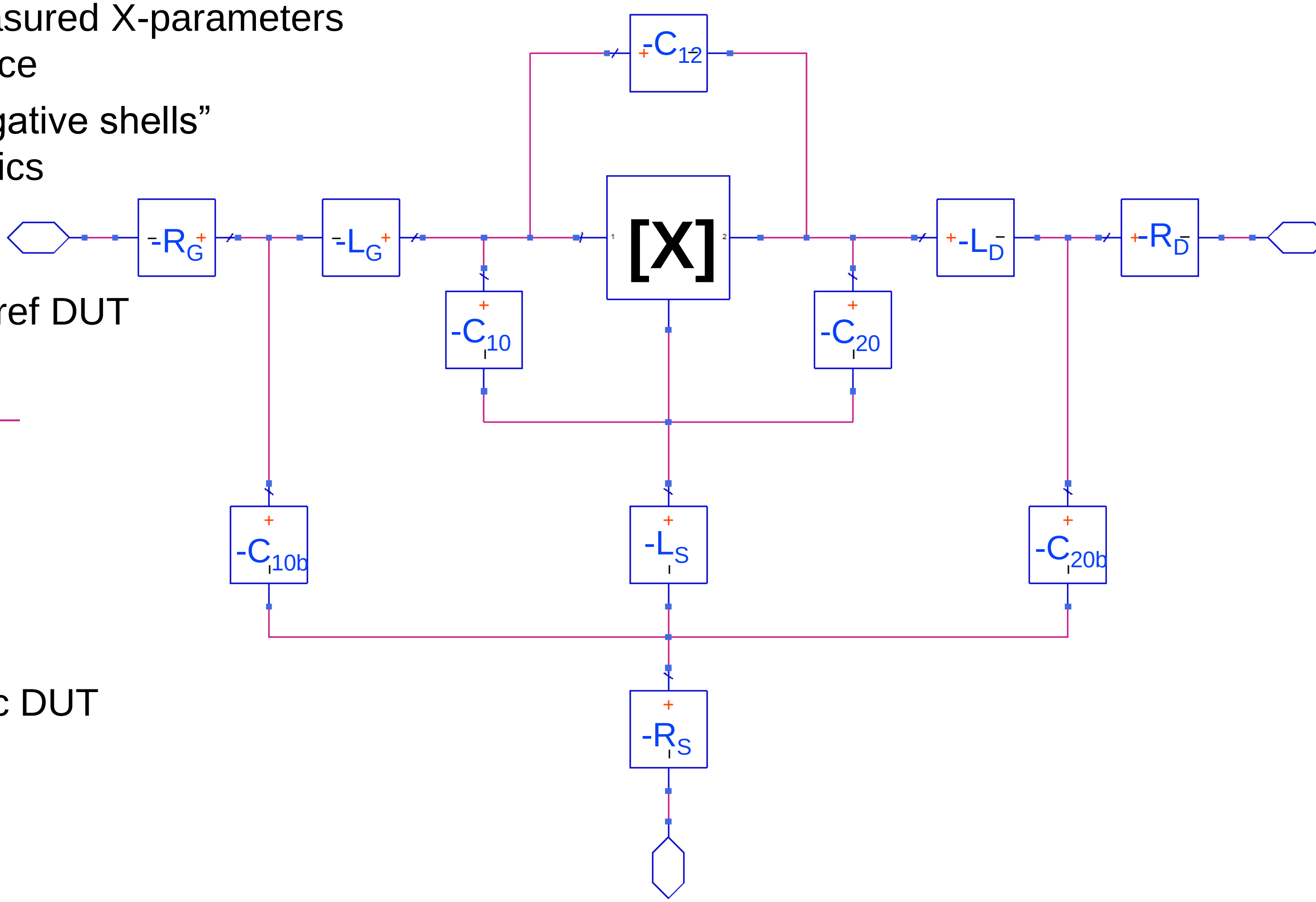
[X]

parasitic element values are assumed known

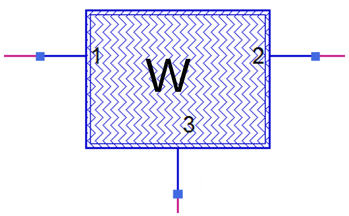
Dealing with parasitic elements

(1) Start with measured X-parameters of reference device

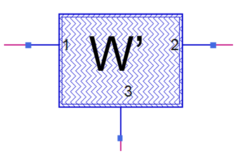
(2) Embed in "negative shells" to remove parasitics



Arrive at intrinsic ref DUT



(3) Scale intrinsic DUT



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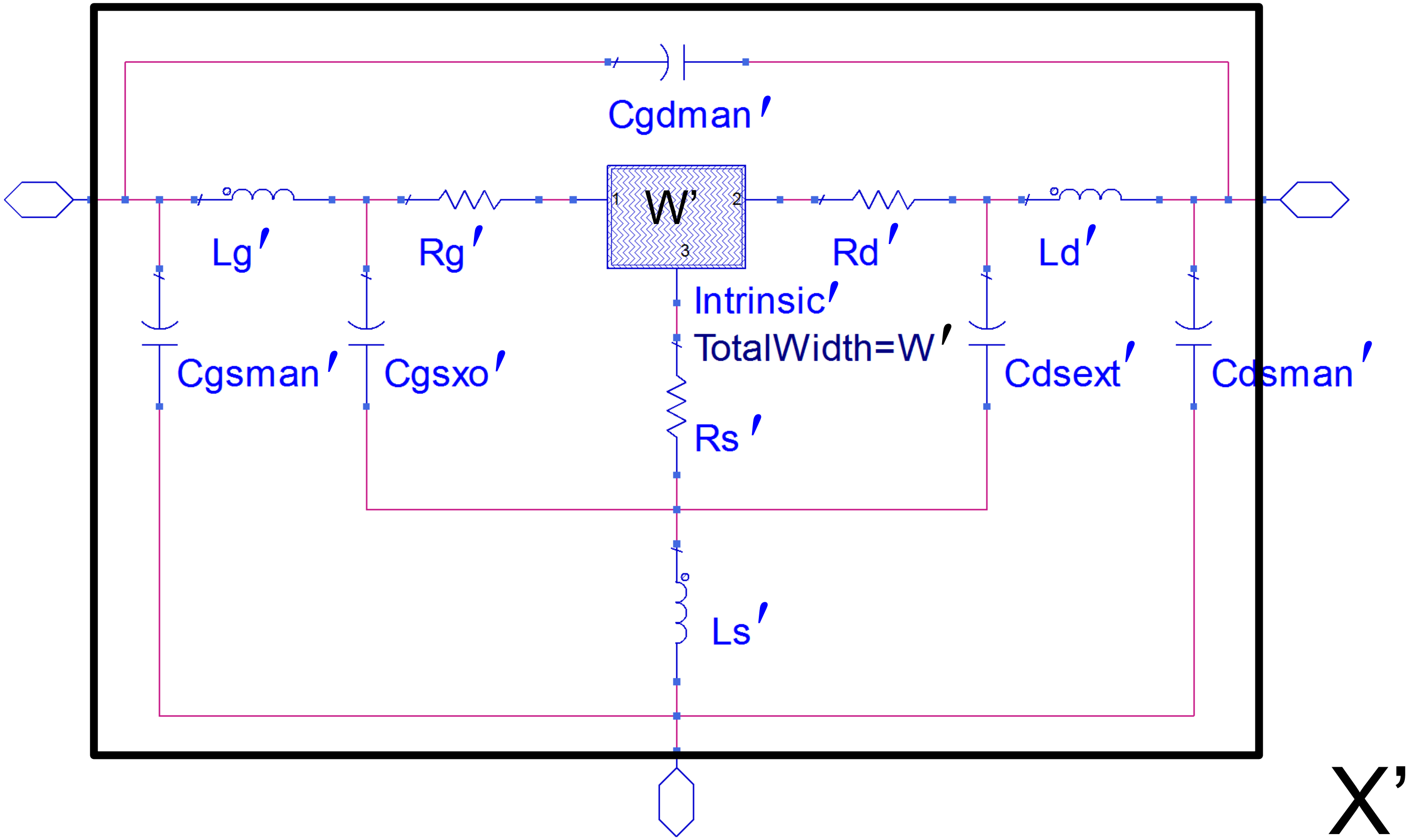
Mathematically Scaling the Waves

(4) Embed scaled parasitics

(5) Re-extract X-parameters



Validation: Experimental

Method

4x60 micron GaAs MESFET (ref. device)

↓

Measure X-parameters on NVNA

↓

Compute B'(A') relationships for 4x30 (scaled) device

↓

Extract X-parameters from scaled B'(A') relations (same Z0)

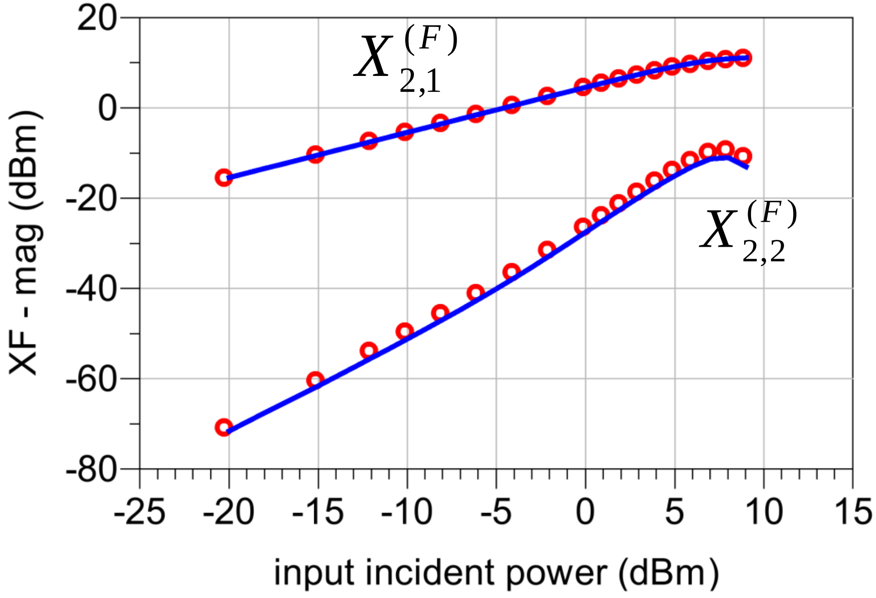
↔ **compare** ↔

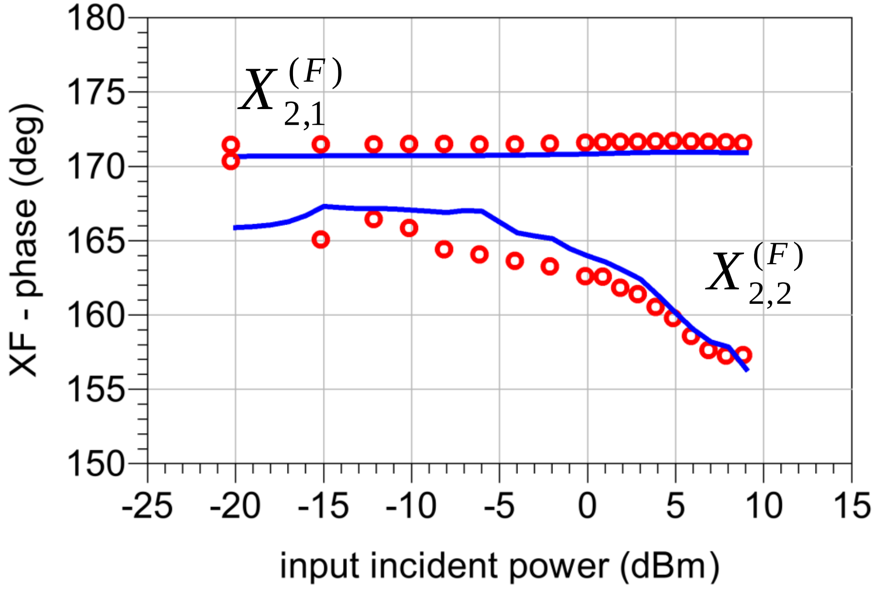
4x30 micron GaAs MESFET (scaled device)

↓

Measure X-parameters on NVNA

Validation: Experimental

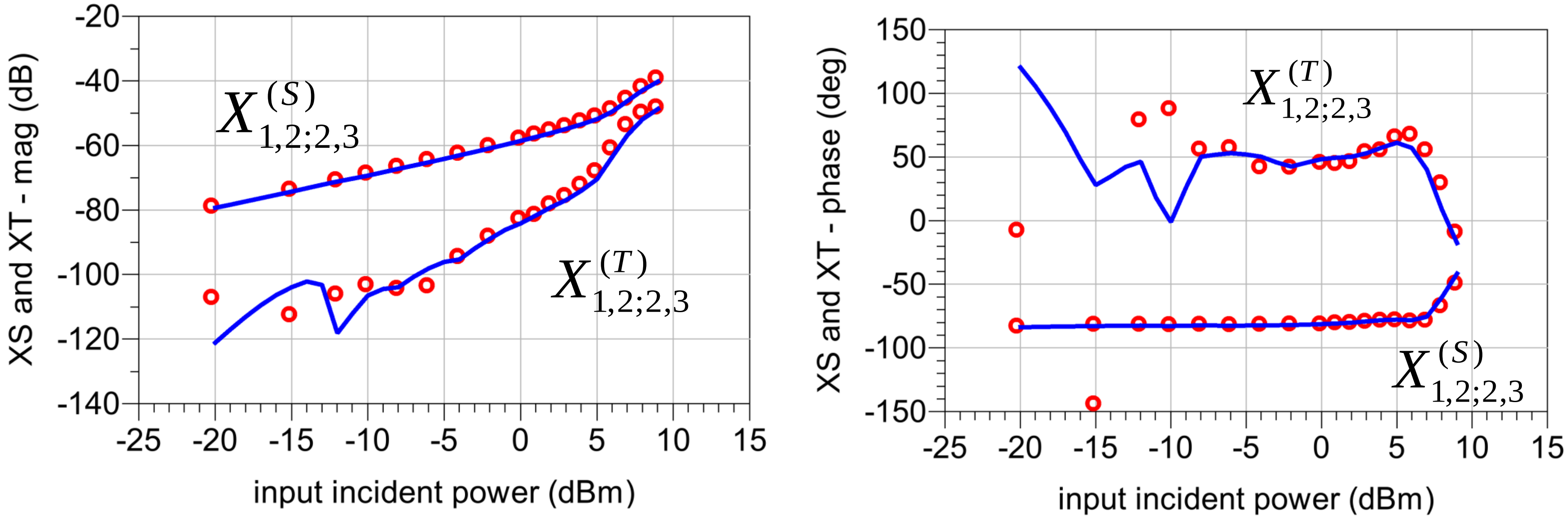




X-parameters measured on device of size 4x30 microns (red symbols)

Measured X-parameters on a 4x60 device mathematically scaled to 4x30 device (blue line)

Validation: Experimental



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↓

Extract X-parameters from scaled B'(A') relations (same Z0)

4x30 micron GaAs MESFET (scaled device)

↓

Measure waveforms

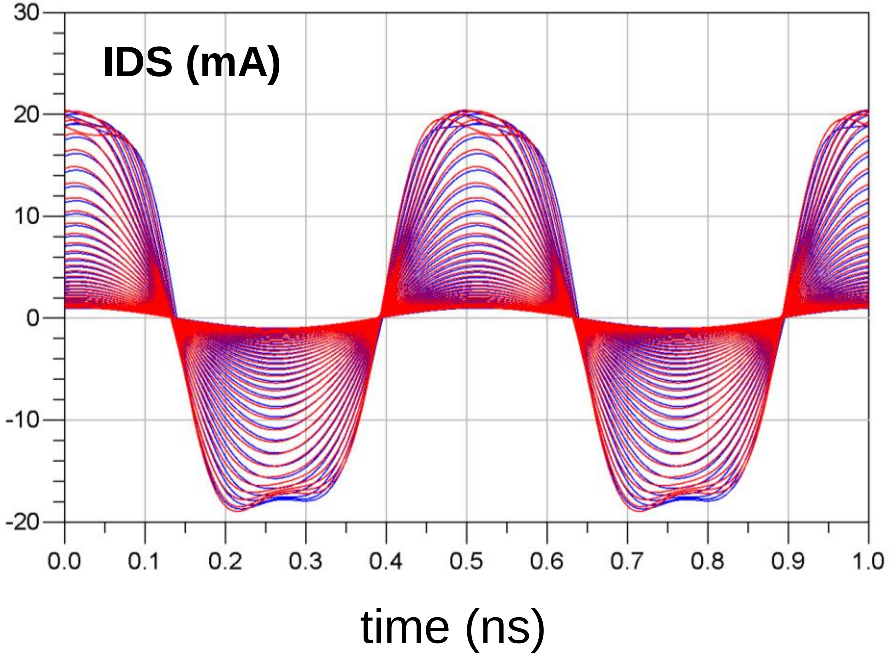
↓

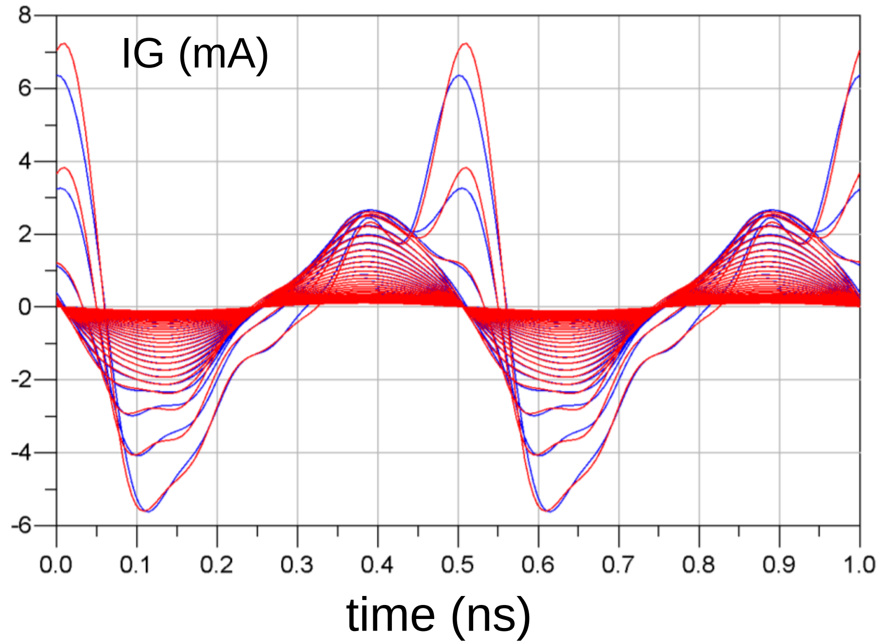
Measure X-parameters on NVNA

compare X-parameters

Validation: Experimental (2)

Scalable FET model from measured X-parameters





From measured X-parameters of complete 4x30 FET

Model scaled to 4x30 starting from X-parameters of 4x60 FET

Summary

- Explicit scaling rules derived for individual X-parameter functions
- Scalable X-parameter transistor model demonstrated
  - A perceived barrier to X-parameters for transistor models is removed
- Extensive nonlinear validation
  - using scalable compact model in simulator
  - using NVNA data on scaled transistors
- X-parameters provide a practical, complementary approach to conventional compact device modeling

Thank You!