Quantum Computing and Simulation

H.-Y. Wong, N. R. Govindugari, A. Zaman, H. Morrell, H. Dhillon, P. Dhillon, K. Beck*, and Y. J. Rosen*

hiuyung.wong@sjsu.edu

Electrical Engineering, San Jose State University, CA, USA

**Lawrence Livermore National Lab*

SAN JOSÉ STATE UNIVERSITY

Agenda

- Self-Introduction
- Overview of Quantum Computing
  - Basics
  - Hardware
- Simulation for Quantum Computers
- Quantum Computing for Simulation
  - HHL Algorithm
  - Variational Quantum Linear Solver

Self-Introduction

- San Jose State University
- Quantum Technology Education
- “Introduction to Quantum Computing: from a Layperson to a Programmer in 30 Steps”

About San Jose State University

36,000 students
Minority Serving Institution (MSI)
Hispanic Serving Institution (HSI)
Electrical Engineering:
~300 master and ~500 undergraduate students

San Jose State tops Stanford and Cal for the most alums now working at Apple, according to LinkedIn's new education search utility.

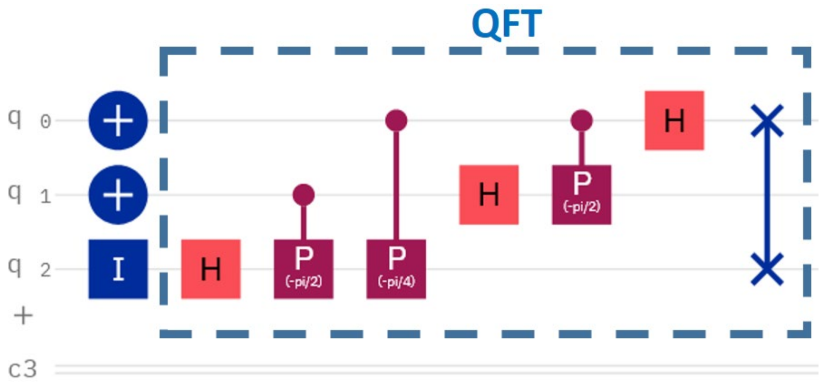
"Our top three are Cisco, Apple and Hewlett Packard," Newell said.

Jobvite, a recruiting platform, found Silicon Valley companies hire more San Jose State alums than any other college or university in the country.

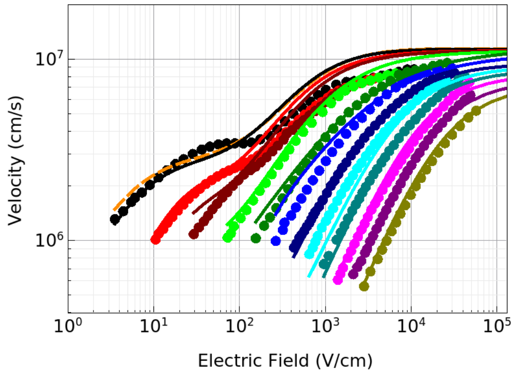
SJSU MSEE Specialization in *Quantum Information and Computing*

- Electrical Engineering, Master of Science
  - EE225 Introduction to Quantum Computing (Every Fall)
  - EE226 Cryogenic Nanoelectronics (Spring 22, every 2 years)
  - EE274 Quantum Computing Architectures (Spring 23, every 2 years)

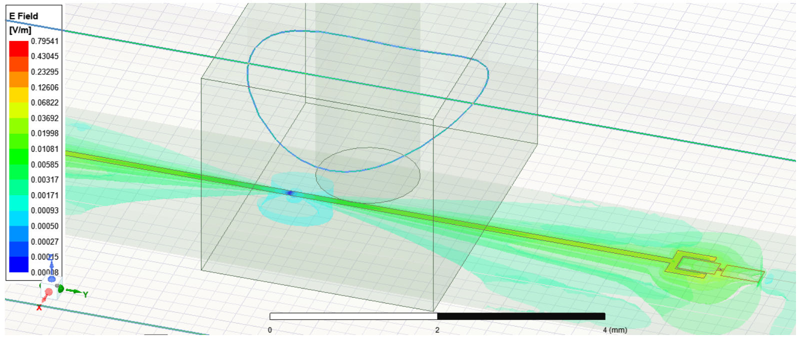
Python, Qiskit,



TCAD, Spectre



HFSS, Qiskit Metal



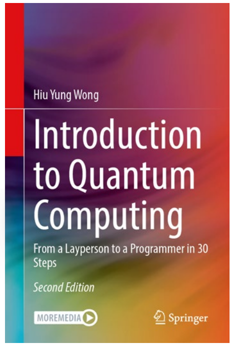
Quantum Technology, Master of Science

- MSQT@SJSU:
  - Start in 2023 Fall
  - Co-housed in Physics and EE
  - Core classes
    - Fundamentals of Quantum Information
    - Quantum Many-Body Physics
    - Quantum Computing Architectures
    - Quantum Programming
- NSF Research Traineeship Program (2125906)
  - Partner with Colorado School of Mines to develop *interdisciplinary* programs
  - Partner with LLNL and industry partners for hands-on experience
- To learn more: https://www.sjsu.edu/quantum/, Email: quantum@sjsu.edu

The degree promotes flexibility by offering a small set of core knowledge courses in quantum fundamentals along with a range of hardware and software focused electives ... partnerships with industry and national labs ... leveraging SJSU's unique position in Silicon Valley.

Resources

- Book: (**2nd Edition with 200+ questions and answers and links to teaching videos)**
  - **Introduction to Quantum Computing: From a Layperson to a Programmer in 30 Steps** | SpringerLink (https://link.springer.com/book/10.1007/978-3-030-98339-0) (Free if your school has a subscription, connect to VPN)
  - **Introduction to Quantum Computing: From a Layperson to a Programmer in 30 Steps**: Wong, Hiu Yung: 9783030983383: Amazon.com: Books (https://www.amazon.com/Introduction-Quantum-Computing-Layperson-Programmer/dp/3030983382)
- Videos (Youtube):
  - **Introduction to Quantum Computing From a Layperson to a Programmer in 30 Steps** – YouTube (https://www.youtube.com/playlist?list=PLnK6MrIqGXsJfcBdppW3CKJ858zR8P4eP)
  - **Quantum Computing Hardware and Architecture – YouTube** (https://www.youtube.com/playlist?list=PLnK6MrIqGXsL1KShnocSdwNSiKnBodpie)



Resources

- IEEE Quantum Week 2023 Tutorial 21:
  - **Introduction to Quantum Computing: From Algorithm to Hardware**
- Download tutorial files and hands-on materials
- https://github.com/hywong2/QCE2023_Tutorial_21
- Recordings are available: https://www.youtube.com/playlist?list=PLnK6MrIqGXsJnZU1SiHhika1QIyDVR41G

Resources

- Hiu Yung Wong, Prabjot Dhillon, Kristin Beck, and Yaniv Jacob Rosen, "A Simulation Methodology for Superconducting Qubit Readout Fidelity," Solid-State Electronics, Volume 201, March 2023, 108582. https://doi.org/10.1016/j.sse.2022.108582
- H. Dhillon, Y. J. Rosen, K. Beck and H. Y. Wong, "Simulation of Single-shot Qubit Readout of a 2-Qubit Superconducting System with Noise Analysis," 2022 IEEE Latin American Electron Devices Conference (LAEDC), 2022, pp. 1-4, doi: 10.1109/LAEDC54796.2022.9908196.
- Hector Morrell and Hiu Yung Wong, "Study of using Quantum Computer to Solve Poisson Equation in Gate Insulators," 2021 International Conference on Simulation of Semiconductor Processes and Devices (SISPAD), 2021, pp. 69-72, doi: 10.1109/SISPAD54002.2021.9592604.
- A. Zaman, Hector Morrell, and Hiu Yung Wong, “A Step-by-Step HHL Algorithm Walkthrough to Enhance Understanding of Critical Quantum Computing Concepts,” in IEEE Access, 2023. 10.1109/ACCESS.2023.3297658

Overview of Quantum Computing

- Introduce the fundamental concepts in quantum computing: State, Superposition, Measurement, Entanglement

Role of Engineers in QC – My perspective

Semiconductor:

Physicist	Engineer
$$\frac{\partial B_{n'\mathbf{K}'}(t)}{\partial t} = -\frac{\mathrm{i}}{\hbar}\left\{\sum_{n\mathbf{K}} M_{nn'}(\mathbf{K},\mathbf{K}')B_{n\mathbf{K}}(t) + E_{B,n'}(\mathbf{K}')\,B_{n'\mathbf{K}'}(t)\right\}$$	$$\mu_{\mathrm{dop}} = \mu_{\mathrm{min1}}\exp\left(-\frac{P_{\mathrm{c}}}{N_{\mathrm{A,0}}+N_{\mathrm{D,0}}}\right)$$
Scattering based on Fermi Golden Rule and Bloch Function (David Esseni, “Nanoscale MOS Transistors“)	TCAD Analytical Equation (Synopsys, Silvaco)
Priceless	**$100M/year**

Quantum Computer:

$$H = \hbar\omega_{\mathrm{r}}\left(a^{\dagger}a + \frac{1}{2}\right) + \frac{\hbar\Omega}{2}\sigma^{z} + \hbar g(a^{\dagger}\sigma^{-} + \sigma^{+}a) + H_{\kappa} + H_{\gamma}.$$

$$\longrightarrow$$

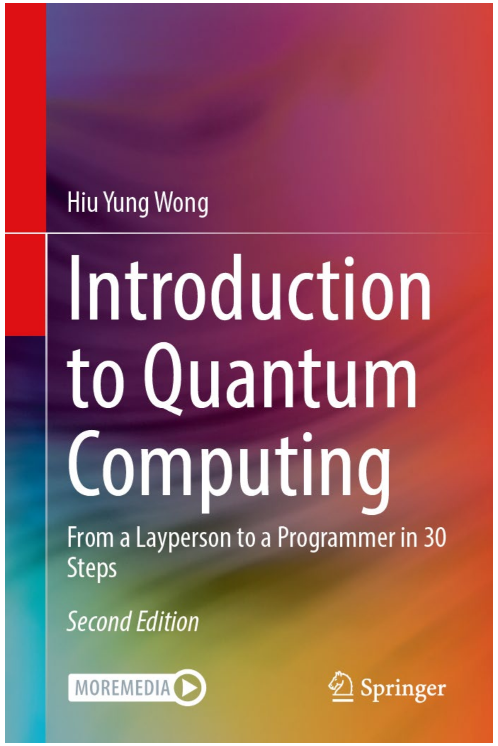
$$$

Jaynes-Cummings Hamiltonian
(Blais et al, PHYSICAL REVIEW A 69, 062320 (2004))

Analytical Equation (Your names, your companies)

There is plenty of room in the (eco-)System

Algorithm



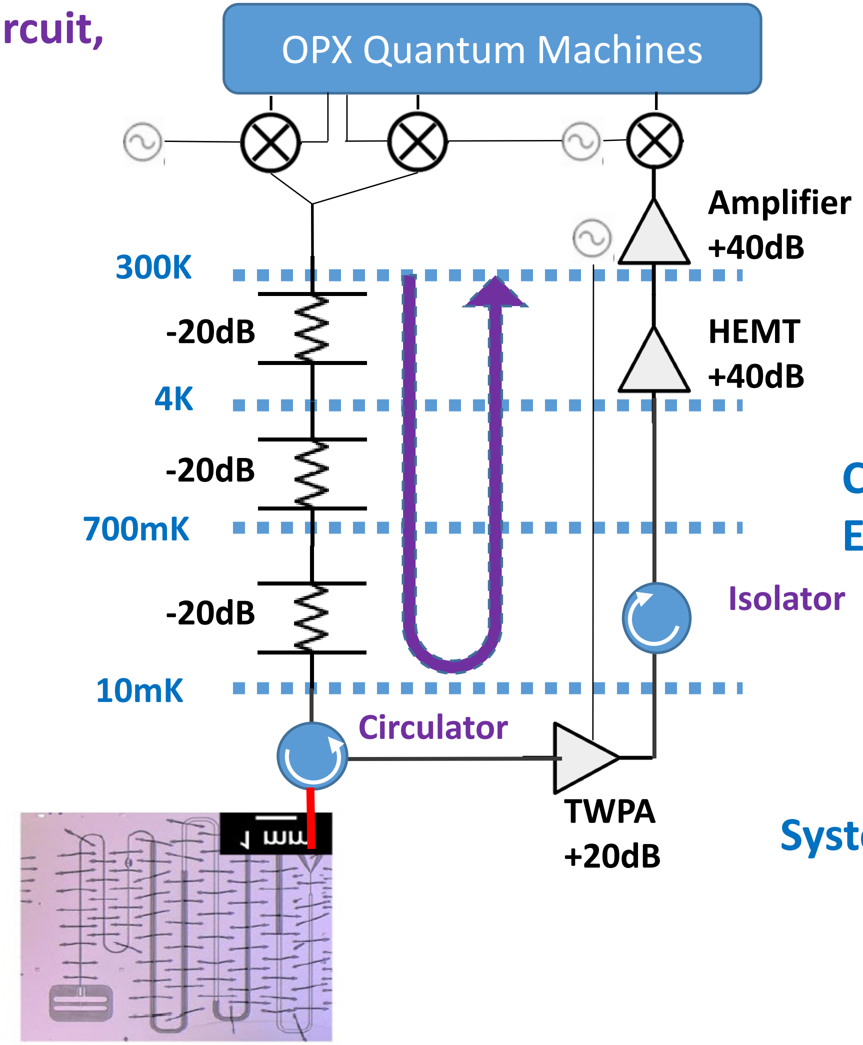
Education

High Speed Circuit, Programming

Microwave Engineering

Vacuum and cryogenic technologies

Qubit Physics

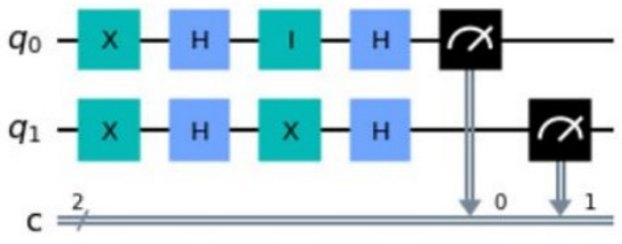


Cryogenic Electronics

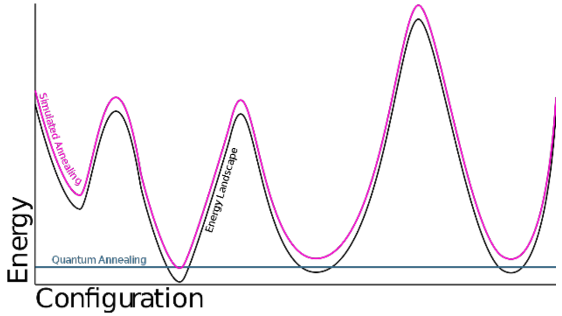
System Modeling

Applications of Quantum Computing

- Quantum computing uses two quantum phenomena
  - Superposition and entanglement and, also, *interference*
- Two major types of quantum computing
  - Gate-based (this talk)
  - Quantum annealing (optimization by minimizing energy)
- Applications
  - Material (battery) and drug (pharma) design
  - Computational Fluid Dynamics
  - Secure communication
  - Quantum machine learning
  - Financial Services and Solutions (e.g. Black Swan Forecasting)



Gate Model



Quantum Annealing

State and Superposition

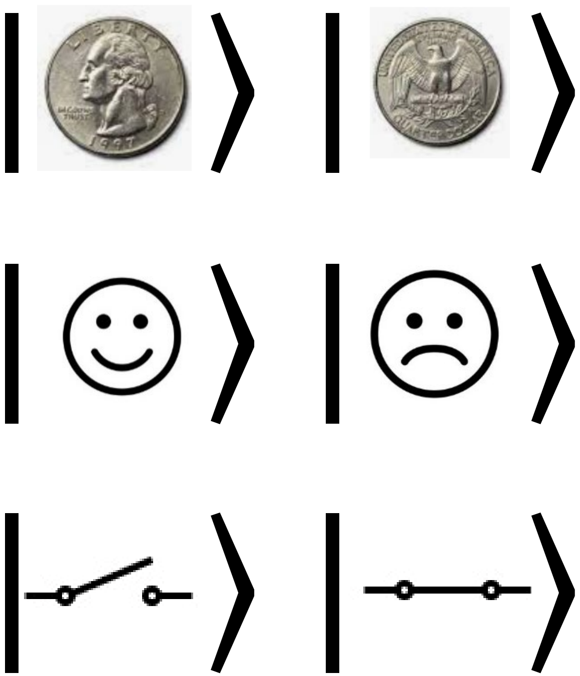
Classical Computing

$0 \qquad 1$

Information represented by the **states**

Quantum Computing (basis states)

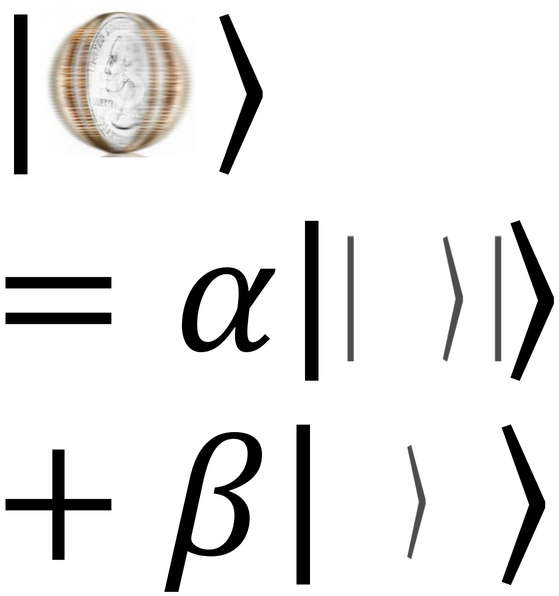
$|0\rangle \qquad |1\rangle$



No difference from classical computing

Quantum Computing with *superposition*

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



Quantum computing is powerful because it uses superposition

Quantum Registers

Value can be stored in a classical register	Basis states in a quantum register
$(0000)_2 = 0$	$\|0\rangle \otimes \|0\rangle \otimes \|0\rangle \otimes \|0\rangle = \|0\rangle\,\|0\rangle\,\|0\rangle\,\|0\rangle = \|0000\rangle = \|0\rangle_{10}$
$(0001)_2 = 1$	$\|0\rangle \otimes \|0\rangle \otimes \|0\rangle \otimes \|1\rangle = \|0\rangle\,\|0\rangle\,\|0\rangle\,\|1\rangle = \|0001\rangle = \|1\rangle_{10}$
$(0010)_2 = 2$	$\|0\rangle \otimes \|0\rangle \otimes \|1\rangle \otimes \|0\rangle = \|0\rangle\,\|0\rangle\,\|1\rangle\,\|0\rangle = \|0010\rangle = \|2\rangle_{10}$
$\vdots$	$\vdots$
$(1111)_2 = 15$	$\|1\rangle \otimes \|1\rangle \otimes \|1\rangle \otimes \|1\rangle = \|1\rangle\,\|1\rangle\,\|1\rangle\,\|1\rangle = \|1111\rangle = \|15\rangle_{10}$

Superposition of basis states of multiple qubits

$$
\begin{aligned}
|\Psi\rangle &= a_0|00\cdots0\rangle + a_1|00\cdots1\rangle + \cdots + a_{2^n-1}|11\cdots1\rangle \\
&= a_0|0\rangle_{10} + a_1|1\rangle_{10} + \cdots + a_{2^n-1}|2^n-1\rangle_{10}
\end{aligned}
$$

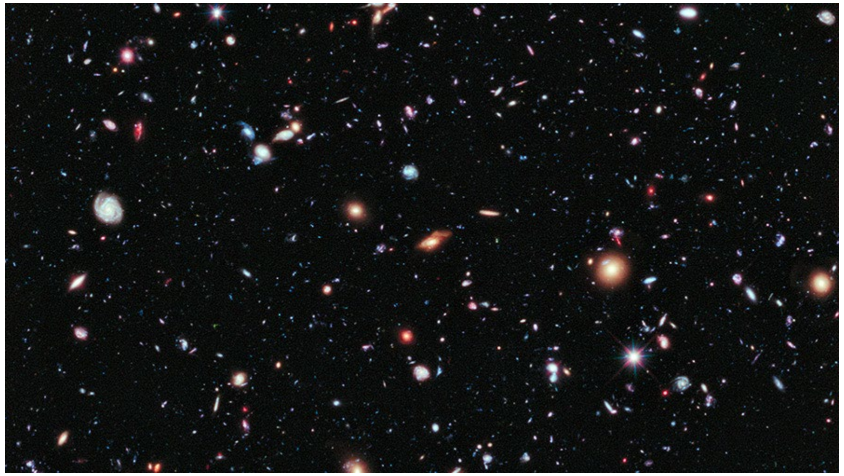
The Power of Superposition

$$|\Psi\rangle = a_0|00\cdots0\rangle + a_1|00\cdots1\rangle + \cdots + a_{2^n-1}|11\cdots1\rangle$$
$$= a_0|0\rangle_{10} + a_1|1\rangle_{10} + \cdots + a_{2^n-1}|2^n-1\rangle_{10}$$

n = 300 (e.g. electrons)

$2^{300} = 10^{90}$ complex coefficients, a_i

Number of atoms in the universe < 10^{82}

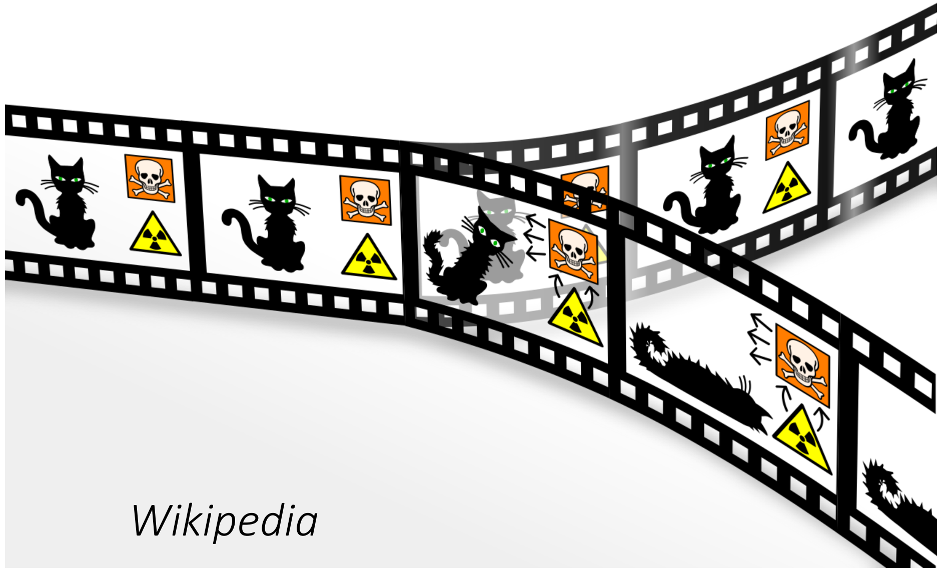


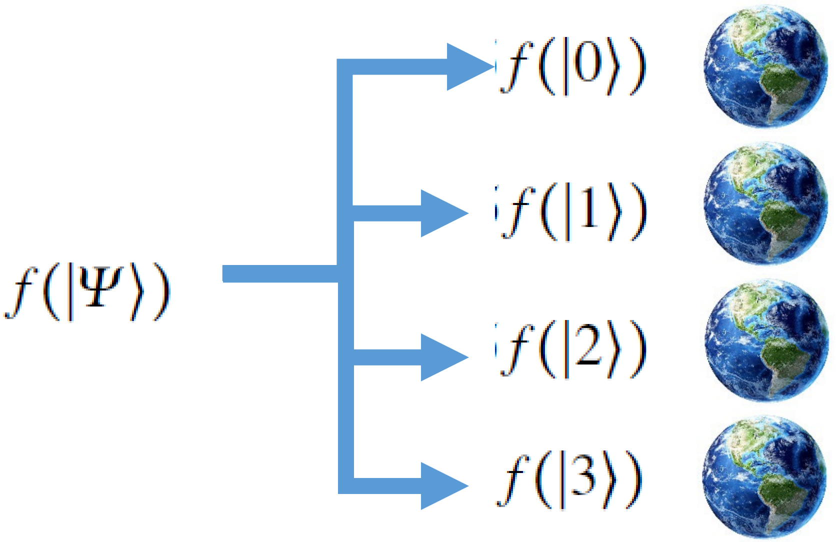
Total number of storage in the world < 10^{21} bytes

Quantum Parallelism

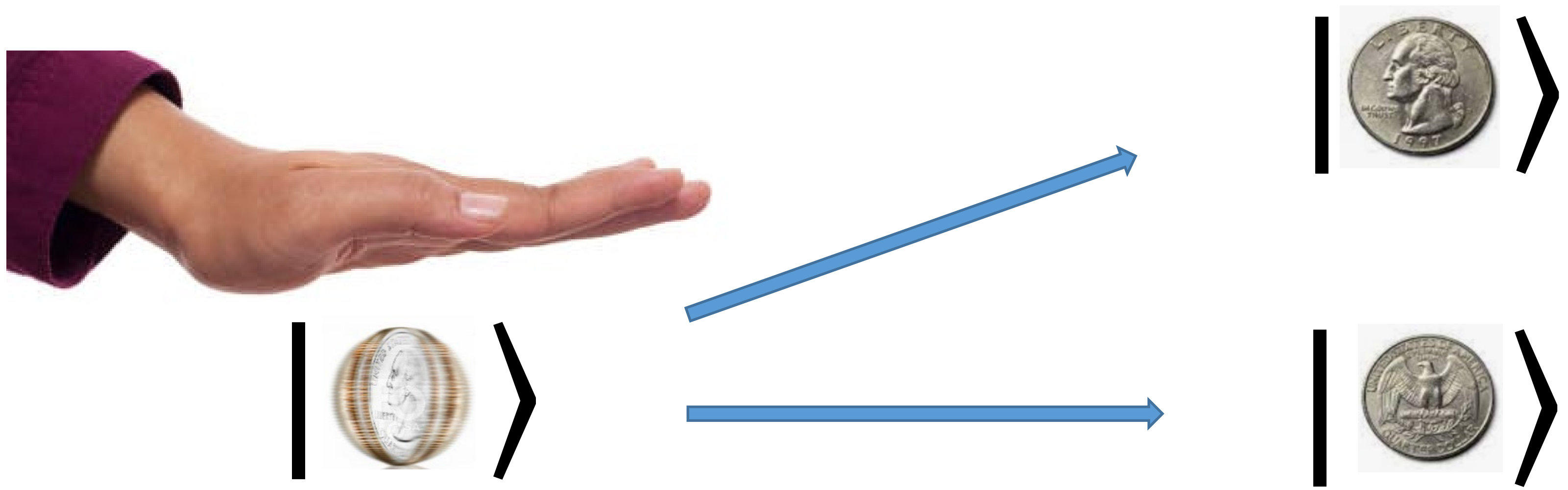
Linear Quantum mechanics

$$f(|\Psi\rangle) = f(0.5|0\rangle + 0.5|1\rangle + 0.5|2\rangle + 0.5|3\rangle)$$
$$= 0.5f(|0\rangle) + 0.5f(|1\rangle) + 0.5f(|2\rangle) + 0.5f(|3\rangle)$$


Wikipedia



Measurement



$$|\psi\rangle = \alpha|0\rangle+\beta|1\rangle$$

$|\alpha|^2$ $\rightarrow$ $|0\rangle$

$|\beta|^2$ $\rightarrow$ $|1\rangle$

Entangled States

Unentangled State

$$|\psi\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) = \underbrace{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)}_{\text{Electron 1}} \otimes \underbrace{\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)}_{\text{Electron 2}}$$

Entangled State: Used in quantum computing algorithms and also quantum communications

$$\left|\Phi^{+}\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \neq |Electron\ 1\rangle \otimes |Electron\ 2\rangle$$

Quantum Entanglement – Spooky Action

Step 1

$$\Phi^+ = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

e^1 e^2

Step 2

e^1 Earth

$$\Phi^+ = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

still entangled

100 light years away e^2

Step 3 measure e^1: 50% to get $|\uparrow\rangle$, assume obtained $|\uparrow\rangle$ for e^1

$$\Phi^+ \xrightarrow{\text{collapse}} |\uparrow\uparrow\rangle$$

e^1 $|\uparrow\rangle$ Earth

100 light years away $|\uparrow\rangle$ e^2

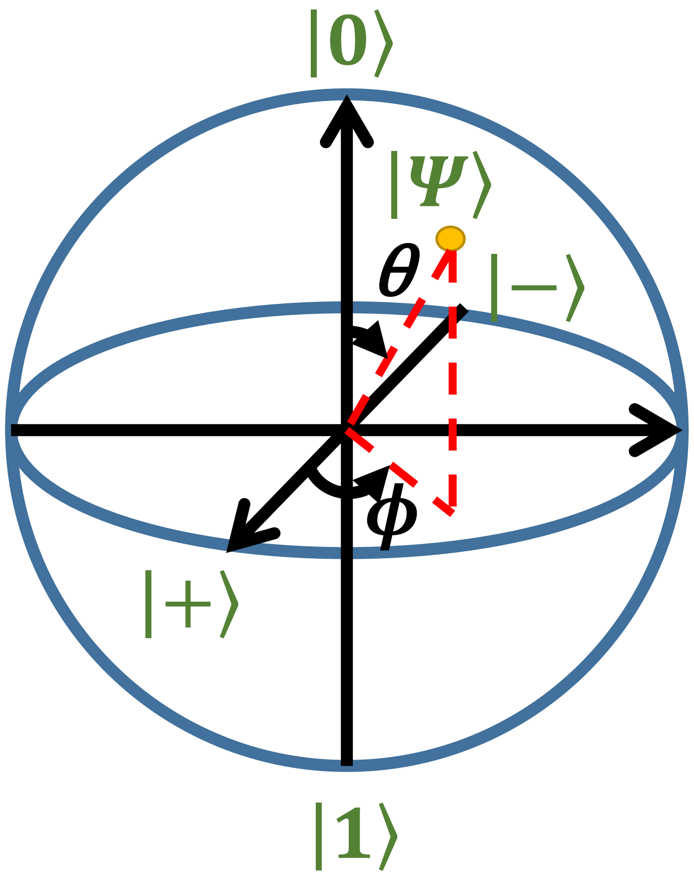
Quantum Gates

- Quantum gates rotate the vector (state) in the corresponding hyperspace
- *Very often, a gate is just a laser or microwave pulse*
- Some gates have classical counterparts
  - NOT (X) gate (1-qubit)
  - XOR (CNOT) gate (2-qubit) $U_{XOR}\,|ab\rangle = |a\,a \oplus b\rangle$
- Some gates have no classical counterparts
  - Hadamard gate (for *Superposition*)

$$\boldsymbol{H}\,|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$\boldsymbol{H}\,|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Bloch Sphere



We embed the hyperspace of a qubit in our real 3D space

Hardware

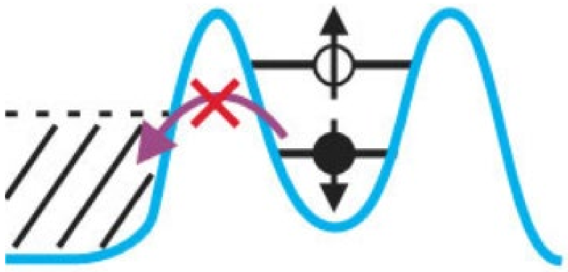
- Understand how a superconducting quantum computer looks like
- Understand how to read a qubit

Implementations of Qubits

$$|Mood\rangle = \alpha|\text{☺}\rangle + \beta|\text{☹}\rangle$$

Not a reliable qubit

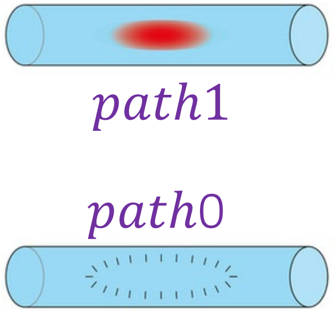
Electron Spin Qubit



$$|\nearrow\rangle = \alpha|\downarrow\rangle + \beta|\uparrow\rangle$$

Physics Today ***72***, 8, 38 (2019)

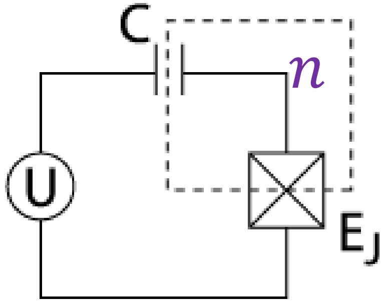
Photonic Qubit



$$|\psi\rangle = \alpha|path0\rangle + \beta|path1\rangle$$

Scientific Reports ***3***, 1394 (2013)

Superconducting Charge Qubit

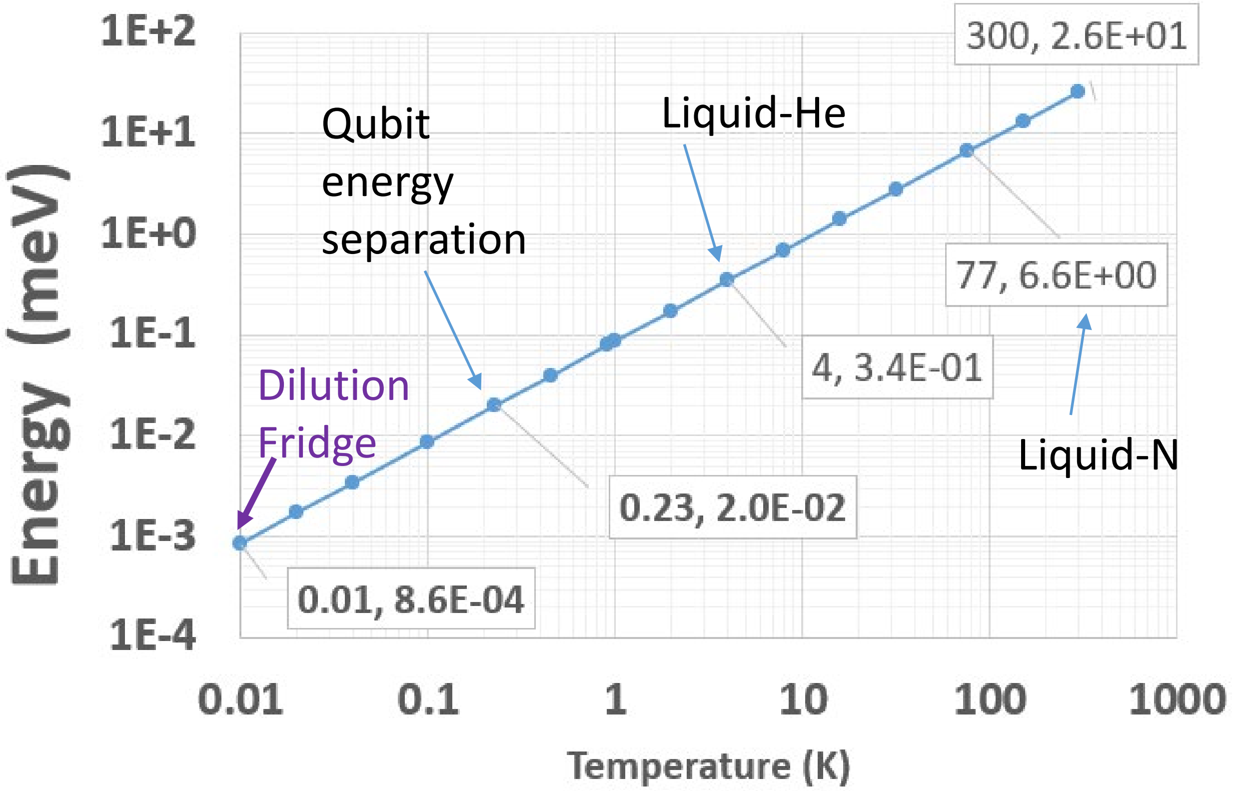


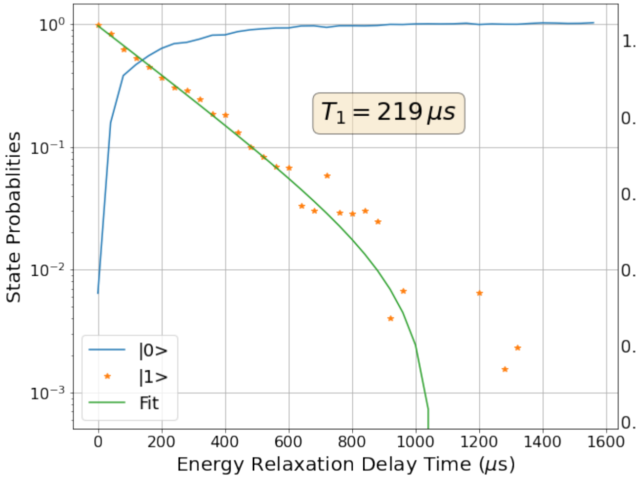
$$|\psi\rangle = \alpha|n=0\rangle + \beta|n=1\rangle$$

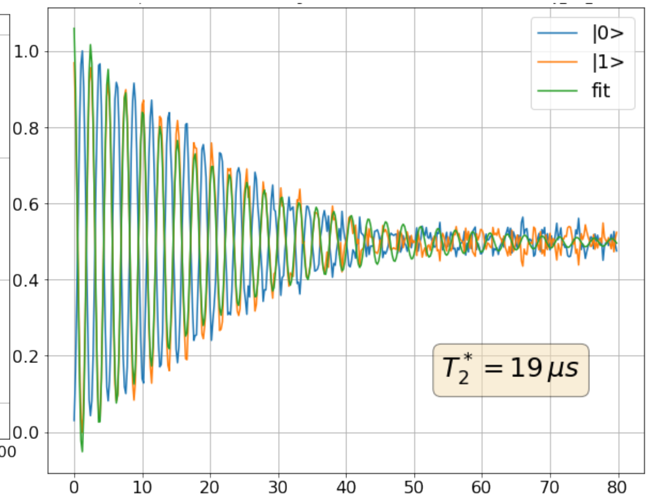
Wikipedia

Noise, De-coherence Time and Energy Scale

- Qubit loses its state due to noise
- Need ultra-low temperature to avoid thermal noise (DR, laser cooling)
- Decoherence time:
  - $T_1 : |1\rangle \Rightarrow |0\rangle$
  - $T_2 : |0\rangle + |1\rangle \Rightarrow ? \, |0\rangle ? \, |1\rangle$

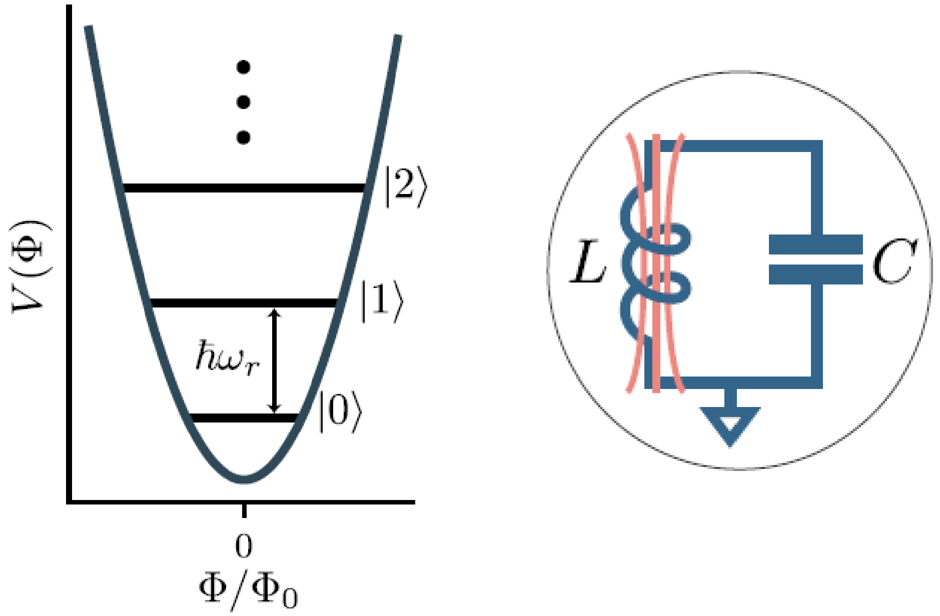






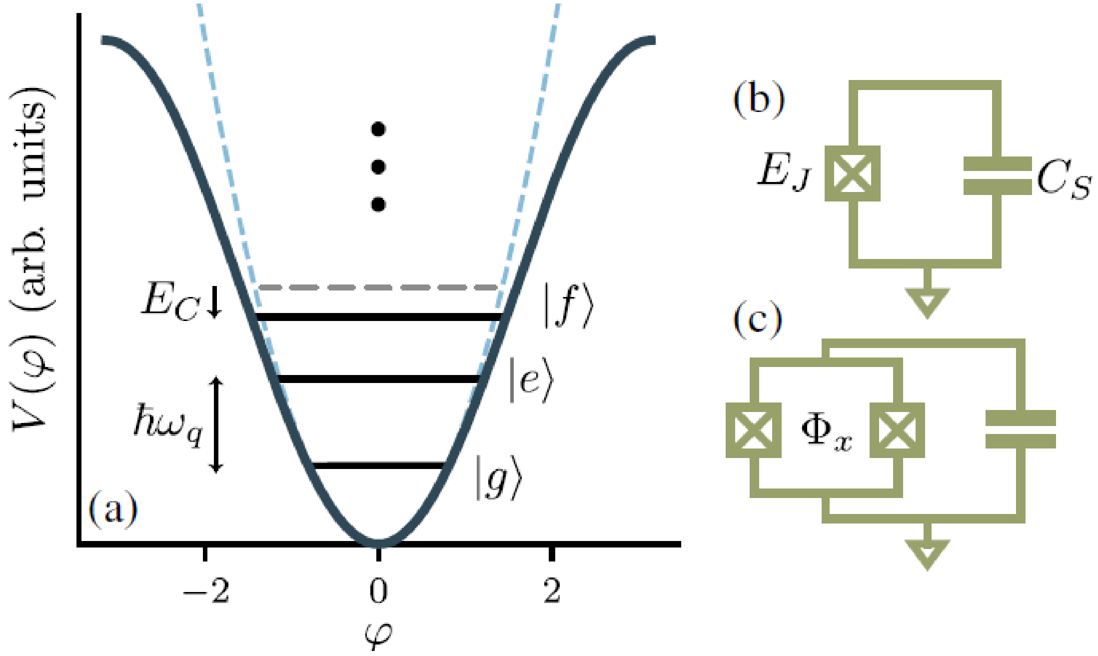
Why not LC Tank?

Blais et al, Review of Modern Physics (2021)



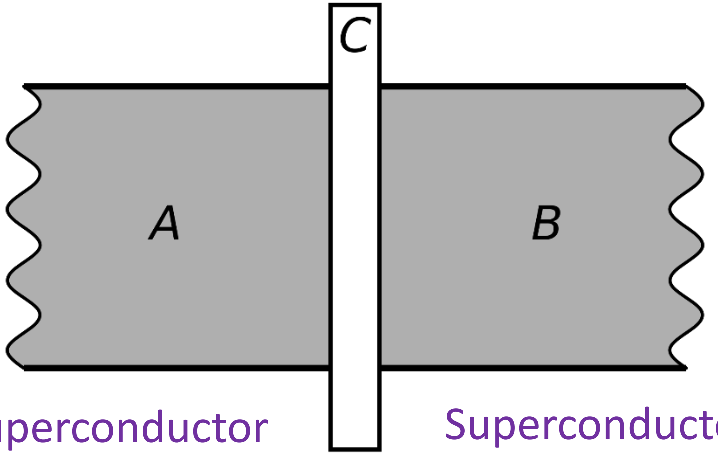
$$H_{LC} = \frac{Q^2}{2C} + \frac{1}{2} C\omega_r^2 \Phi^2$$ Lack of anharmonicity

Generalized momentum: Q
Generalized Coordinate: Φ



- Charge qubit
- Transmon qubit when $E_C << E_J$ (less sensitive to charge noise, n_g)

Josephson Junction – Non-Linear Inductor



Superconductor
Al

Superconductor
Al

Insulator
(Al_2O_3)

Wiki

Josephson Equations

$$I(t) = I_c \sin(\varphi(t))$$

$$\frac{\partial \varphi}{\partial t} = \frac{2eV(t)}{\hbar}$$

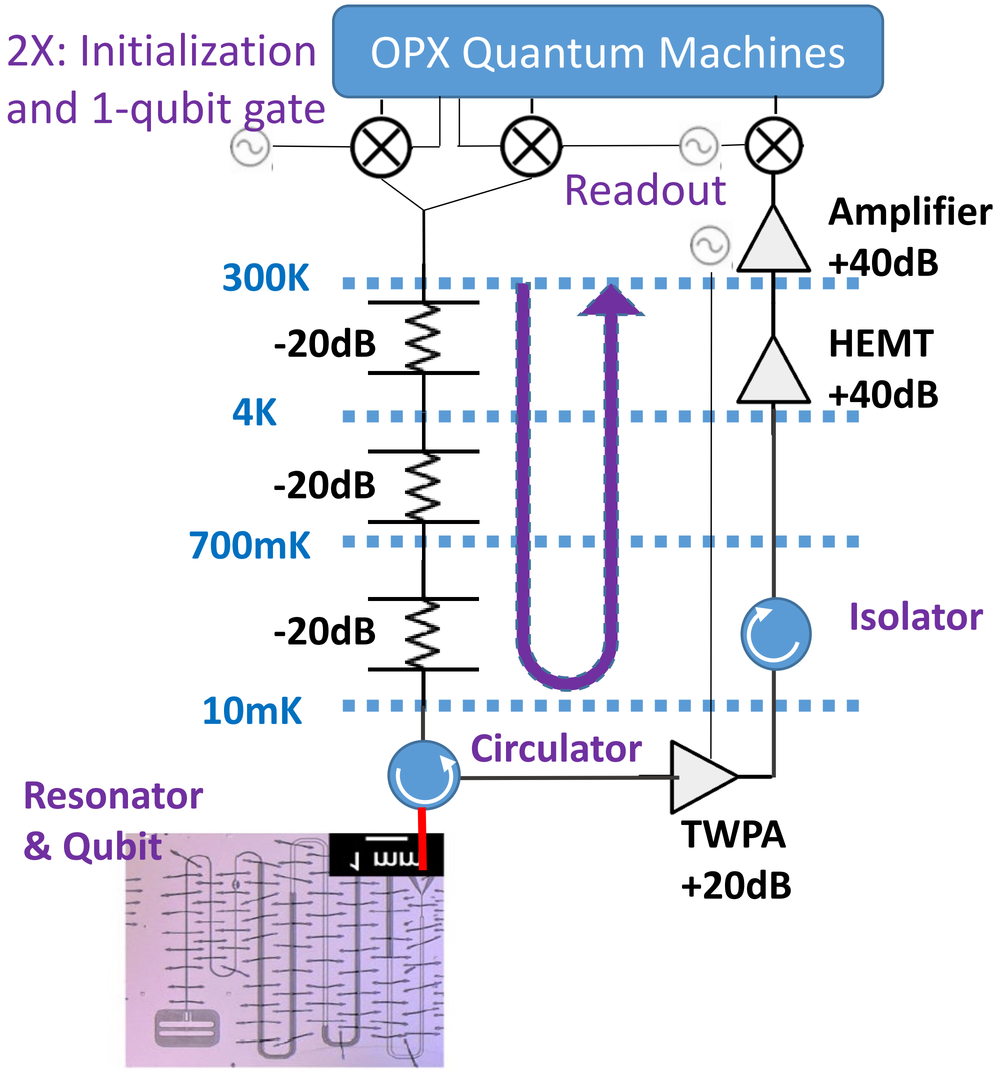
Nonlinear Inductance

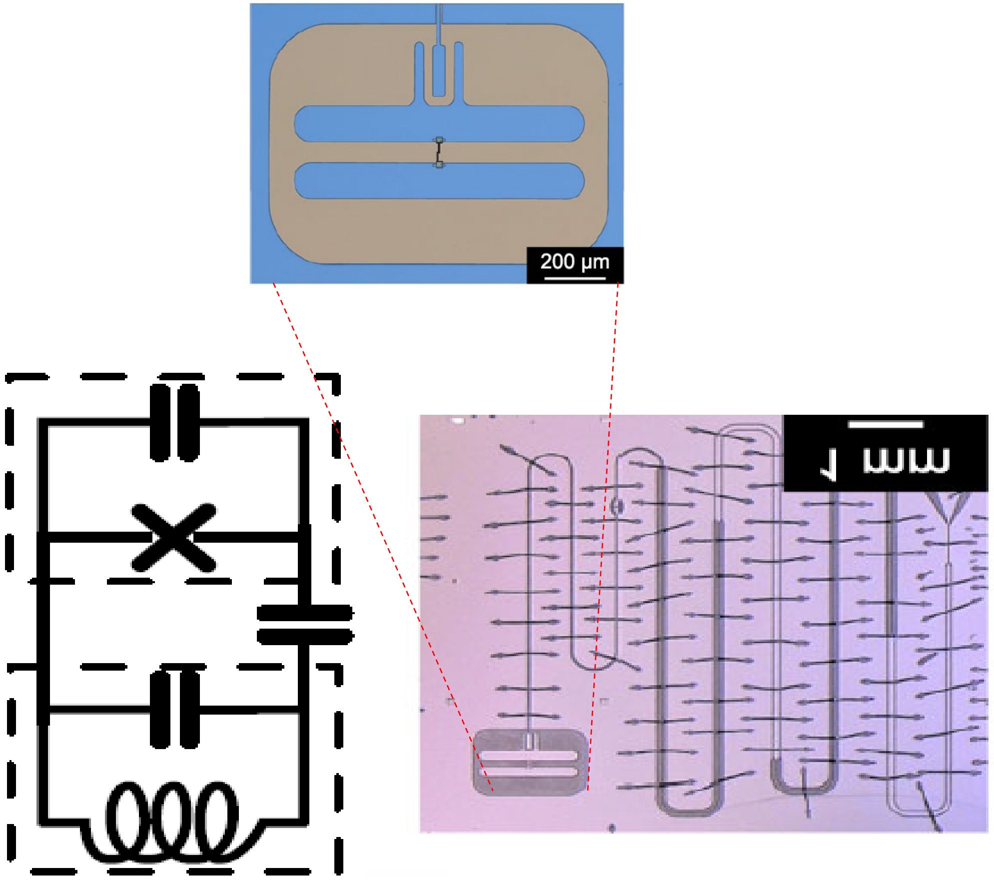
$$L(\varphi) = \frac{\Phi_0}{2\pi I_c \cos\varphi} = \frac{L_J}{\cos\varphi}.$$

Josephson Energy

$$E(\varphi) = -\frac{\Phi_0 I_c}{2\pi} \cos\varphi = -E_J \cos\varphi$$

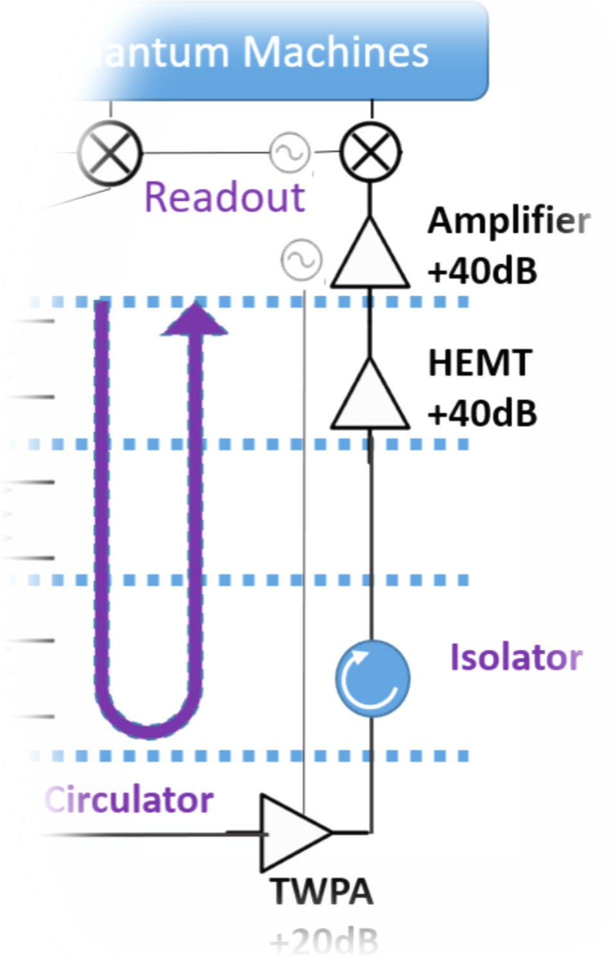
System Overview of a Superconducting Quantum Computer





A. Place, et al., Nat. Comm 12, 1779 (2021)

Signal Amplification and Noise



Friis' Equation

$$NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{P1}} + \cdots + \frac{NF_m - 1}{A_{P1} \cdots A_{P(m-1)}}$$

Reminder:

$$\mathrm{NF}|_{\mathrm{dB}} = 10 \log \frac{SNR_{in}}{SNR_{out}}.$$

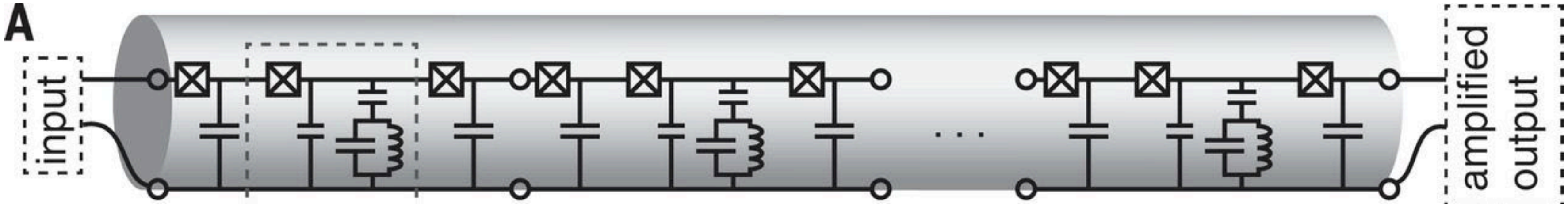
$$P(dB) = 10 \log_{10} \frac{P(W)}{1W} \quad P(dBm) = 10 \log_{10} \frac{P(W)}{1mW}$$

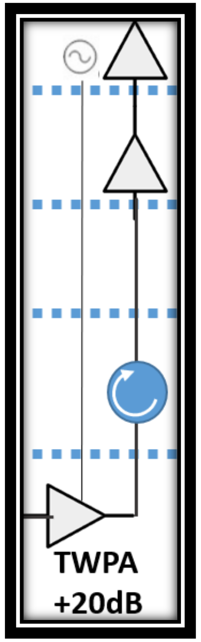
$$G(dB) = 20 \log_{10} G$$

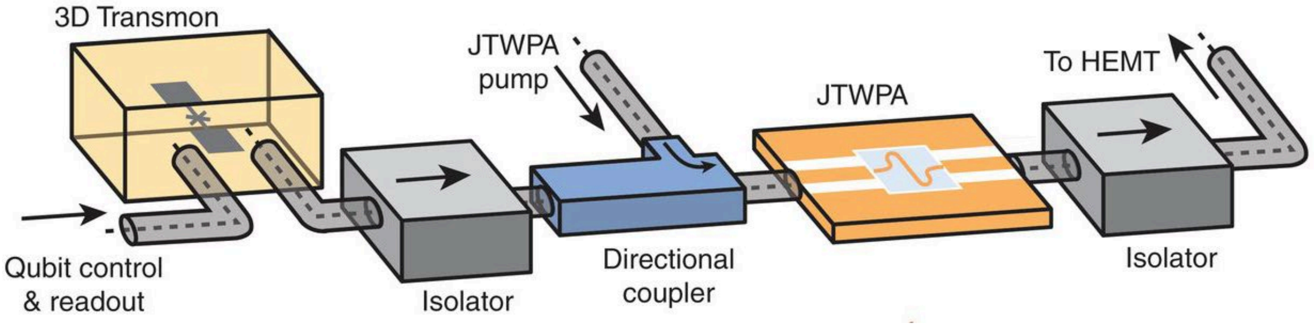
Numerical Example:

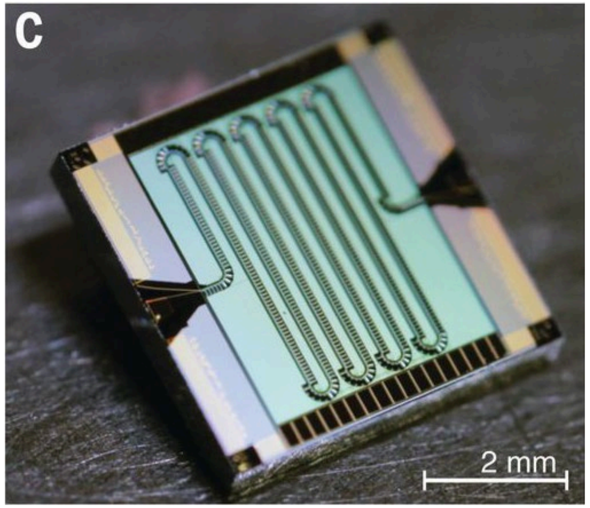
T	Signal (dBm)	Noise (dBm)	SNR (dB)
0.01	-9.28E+01	-1.03E+02	9.901334E+00
1.5	-5.28E+01	-6.27E+01	9.896553E+00
54	-1.28E+01	-2.27E+01	9.896535E+00

Traveling Wave Parametric Amplifier





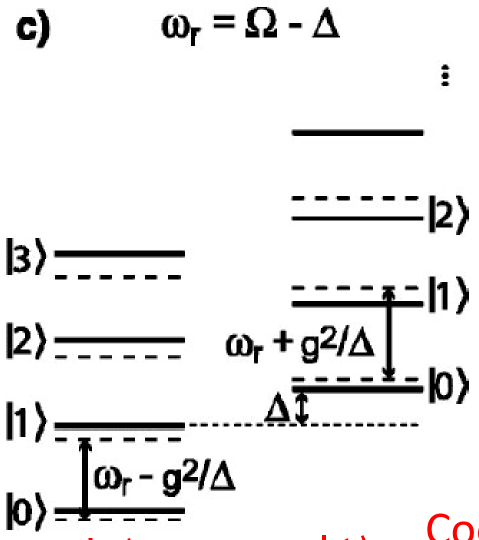




Add minimum noise to the signal (Quantum Limited Amplifier)
Noise source: Uncertainty Principle

C. Macklin et al., "A near–quantum-limited Josephson traveling-wave parametric amplifier", Science, 2015

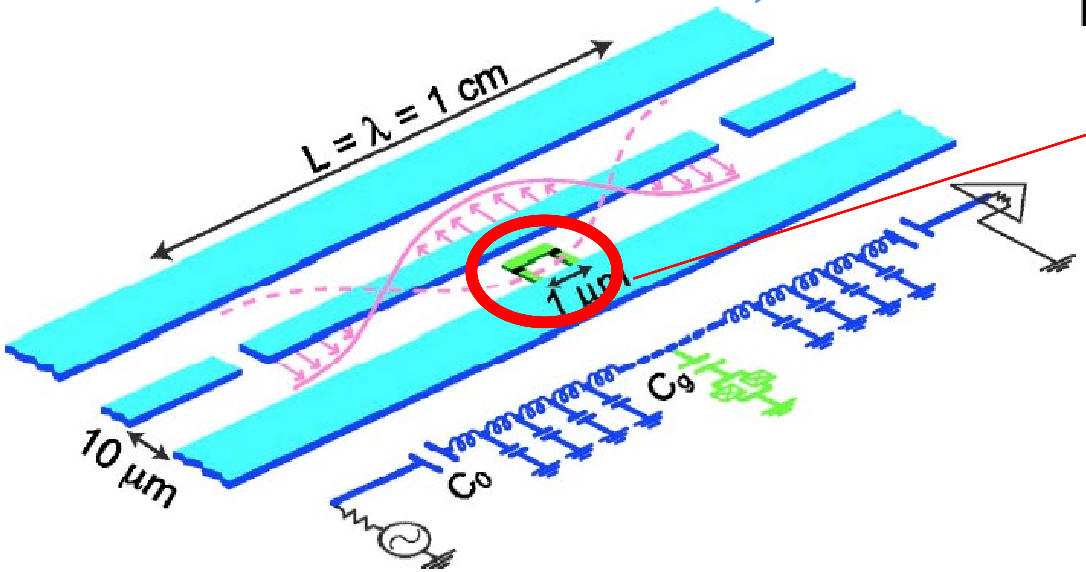
Circuit QED



Resonator as Cavity

$|0\rangle$ $|1\rangle$

Cooper Pair Box:
Artificial Atom



$|0\rangle_{ph}|0\rangle_{charge}$

$\omega_r - \frac{g^2}{\Delta}$

$|1\rangle_{ph}|0\rangle_{charge}$

Apply a
pulse

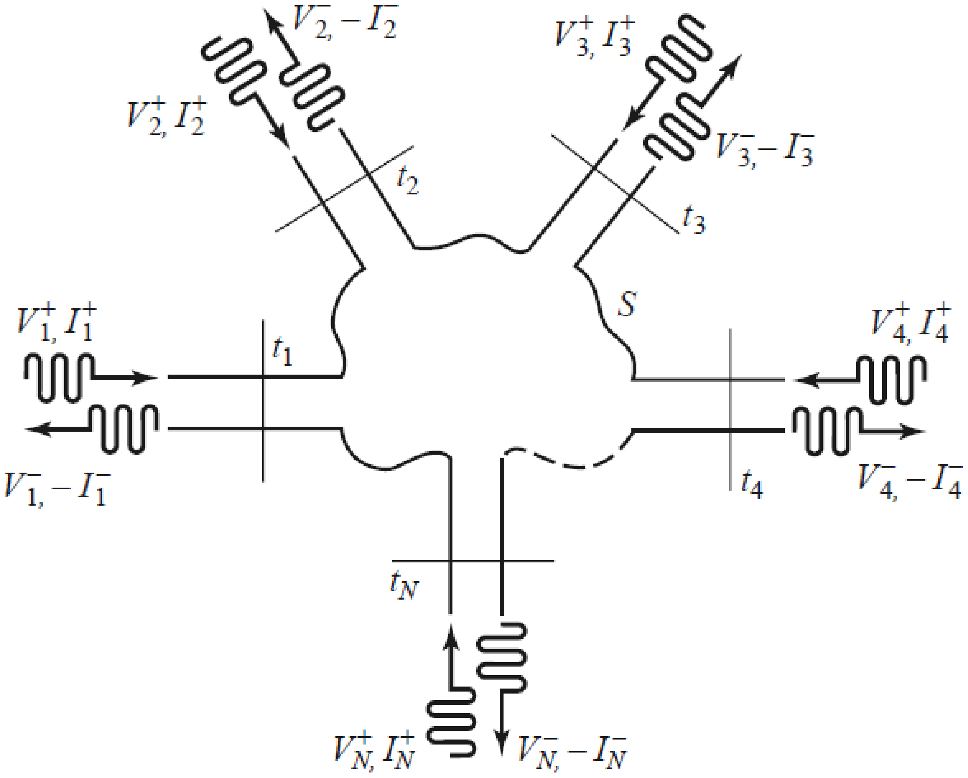
$|0\rangle_{ph}|1\rangle_{charge}$

$\omega_r + \frac{g^2}{\Delta}$

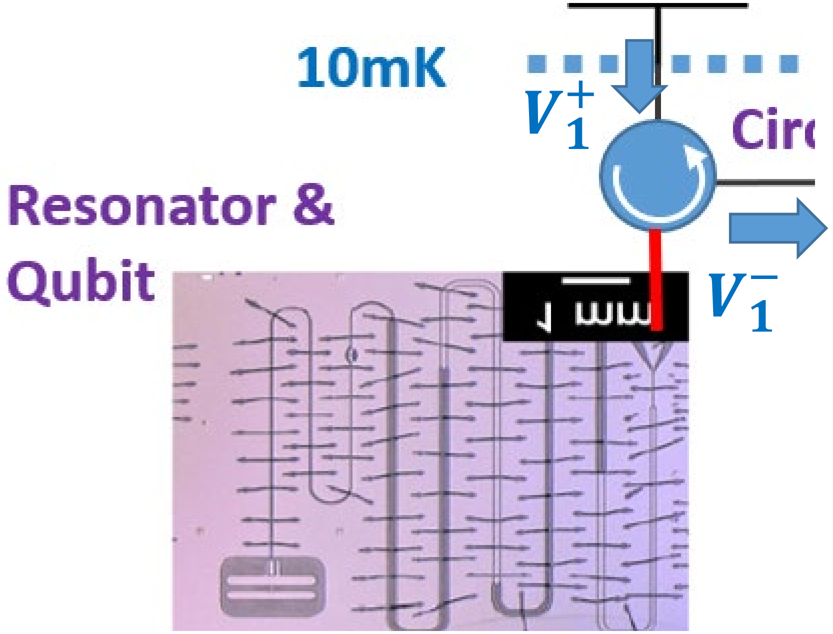
$|1\rangle_{ph}|1\rangle_{charge}$

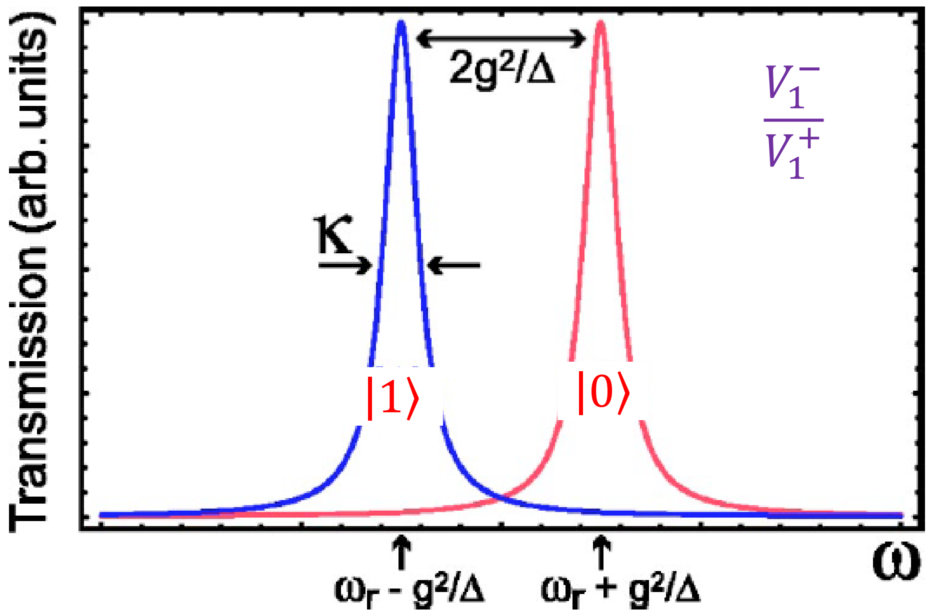
(Blais et al, PHYSICAL REVIEW A 69, 062320 (2004))

Scattering Matrix



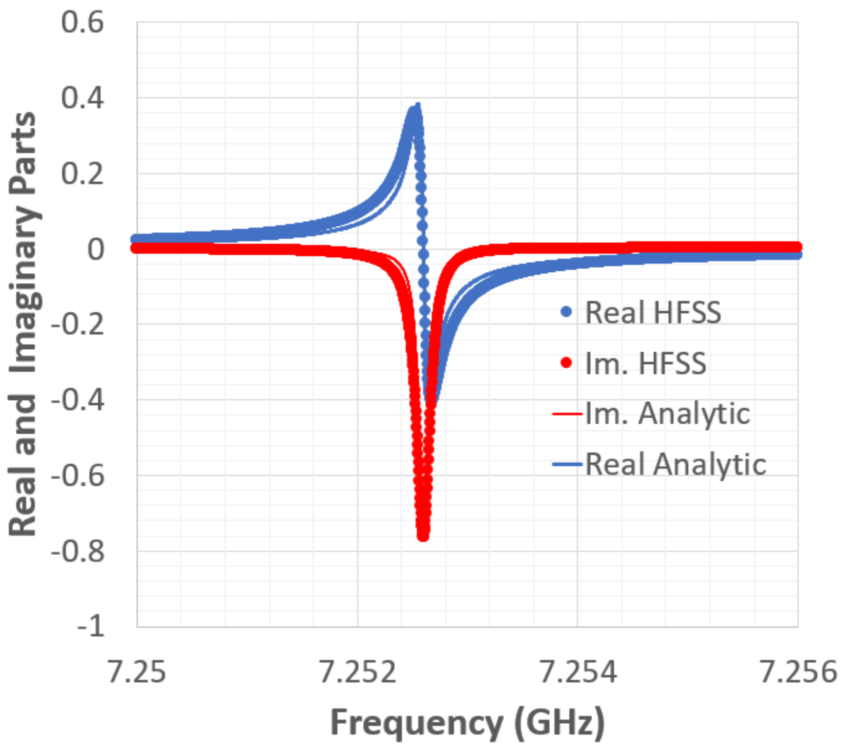
Pozar's Book

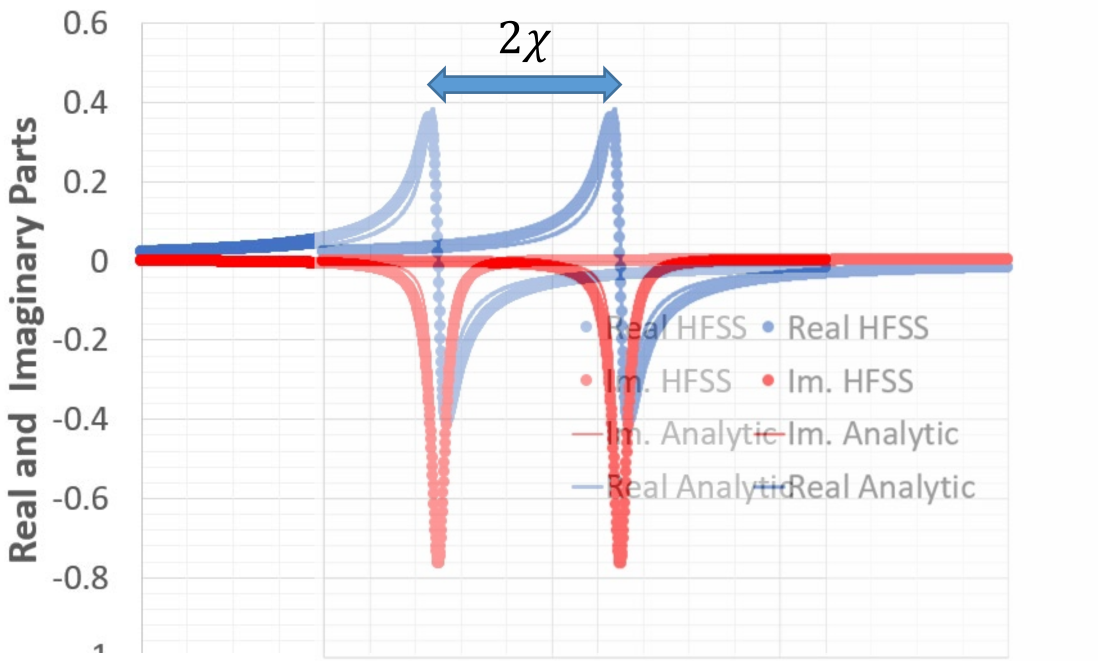




Phys. Rev. A 69, 062320 (2004)

Scattering Matrix



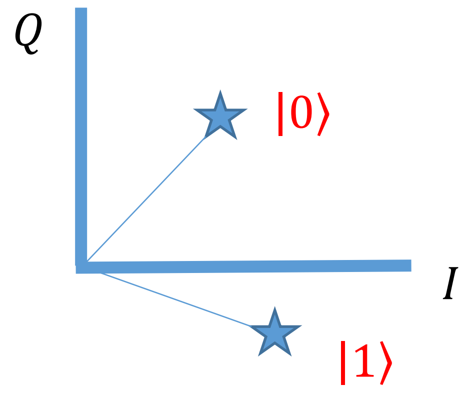


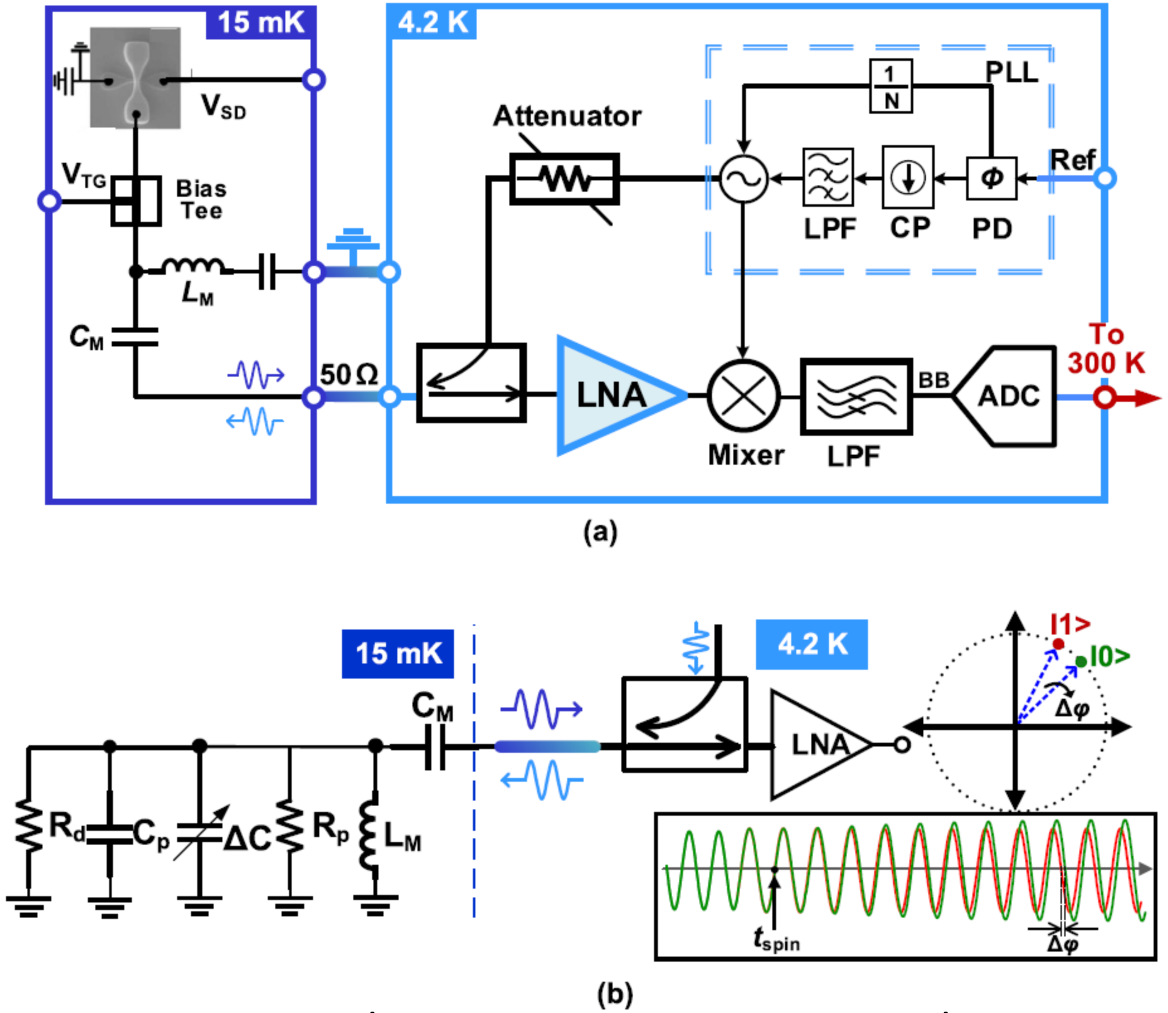
IQ Signal for Reading

Si QDOT qubit reading

$$A\cos(2\pi f t+\phi)$$
$$= A\cos(2\pi f t)\cos(\phi)\text{-}A\sin(2\pi f t)\sin(\phi)$$
$$= A\cos(\phi)\cos(2\pi f t)\text{-}A\sin(\phi)\sin(2\pi f t)$$
$$= I\cos(2\pi f t)\text{-}Q\sin(2\pi f t)$$

$$\tan(\phi) = -\frac{Q}{I}$$



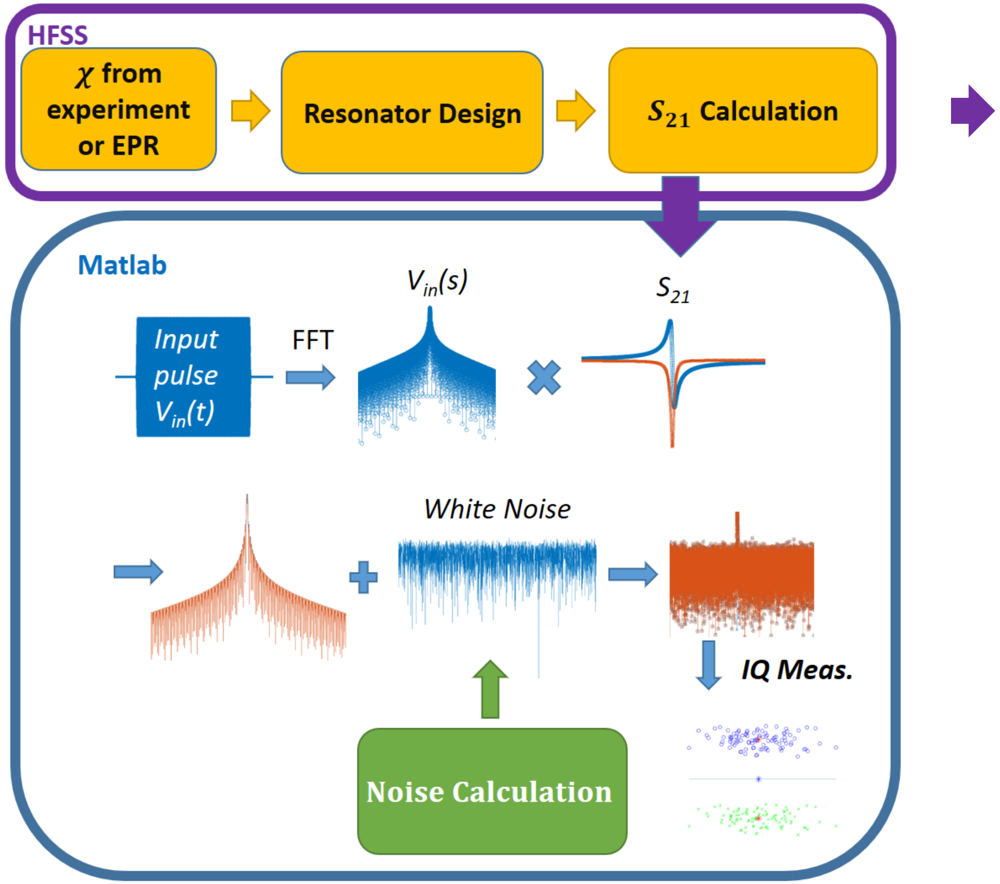


IEEE JSSC, vol. 56, no. 7, pp. 2040-2053, July 2021

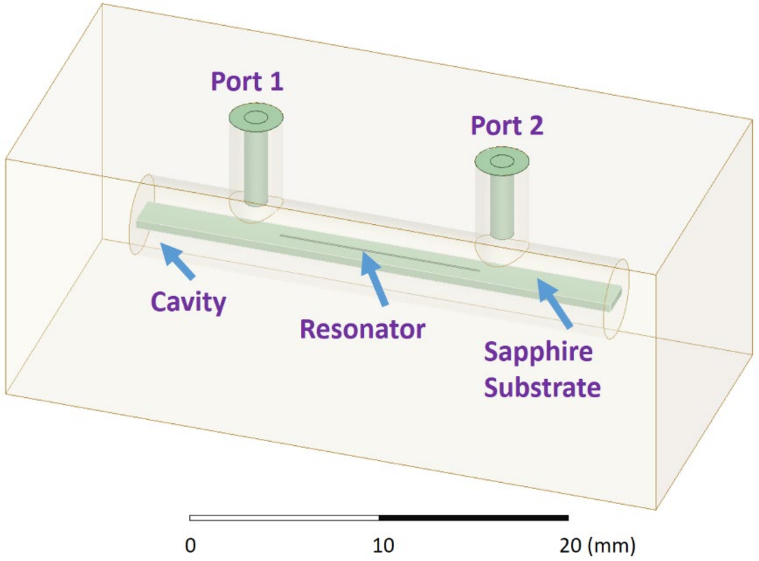
Simulation for Quantum Computers

- Hiu Yung Wong, Prabjot Dhillon, Kristin Beck, and Yaniv Jacob Rosen, "A Simulation Methodology for Superconducting Qubit Readout Fidelity," Solid-State Electronics, Volume 201, March 2023, 108582. https://doi.org/10.1016/j.sse.2022.108582
- H. Dhillon, Y. J. Rosen, K. Beck and H. Y. Wong, "Simulation of Single-shot Qubit Readout of a 2-Qubit Superconducting System with Noise Analysis," 2022 IEEE Latin American Electron Devices Conference (LAEDC), 2022, pp. 1-4, doi: 10.1109/LAEDC54796.2022.9908196.

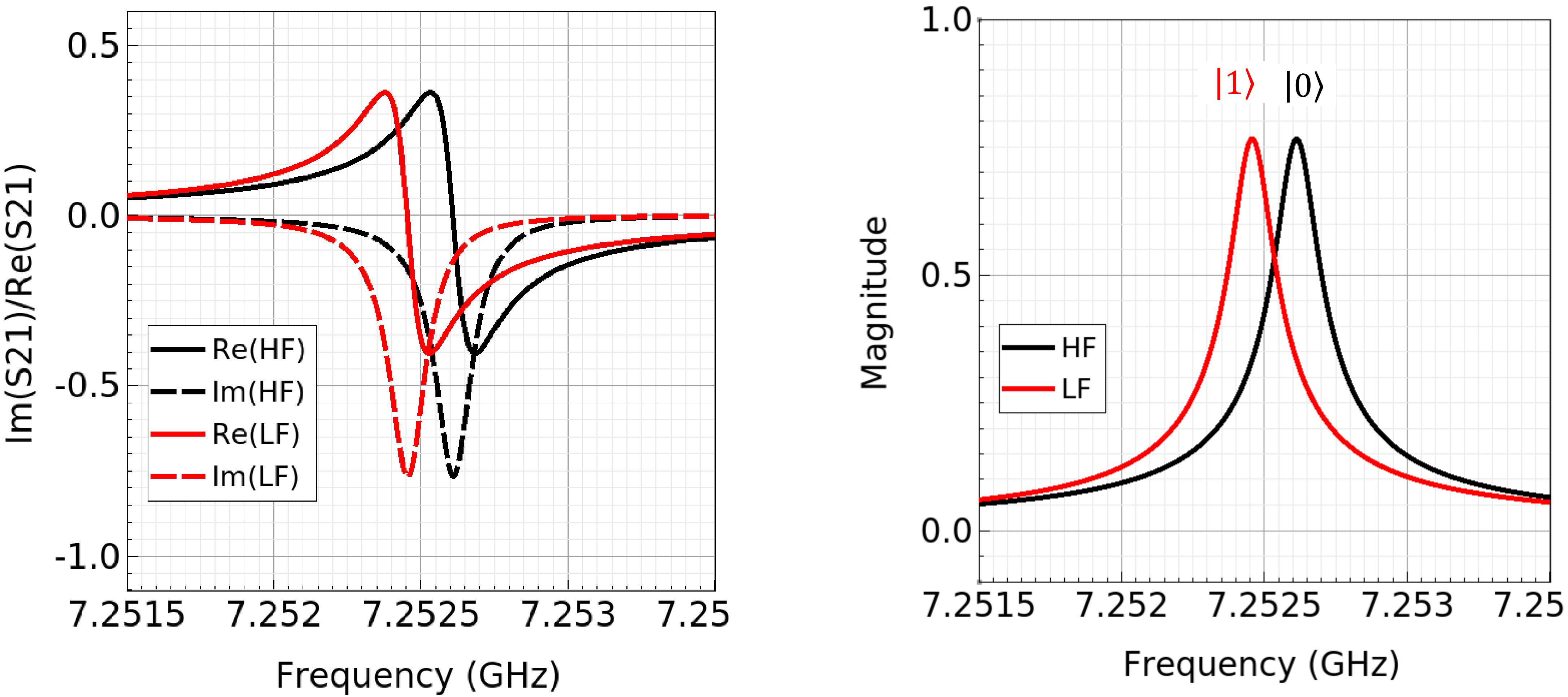
Simulation Methodology – Cross Kerr



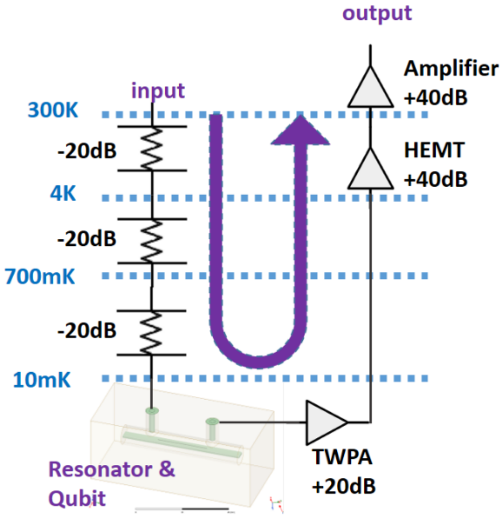
- χ = 114kHz (from experiment)
- Design 2 resonators:
  - 7.252456GHz ($|0\rangle$)
  - 7.252612GHz ($|1\rangle$)
  - Q=47k
  - Dense mesh required

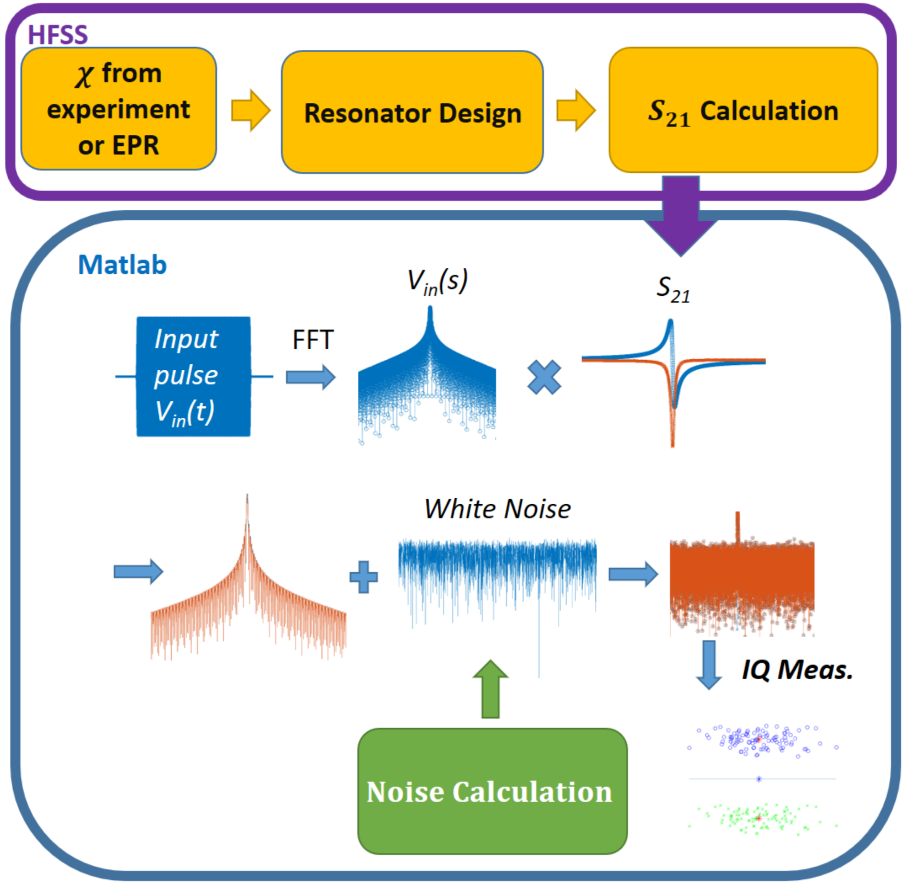


S21


0.5
0.0
-0.5
-1.0
Im(S21)/Re(S21)
Re(HF)
Im(HF)
Re(LF)
Im(LF)
7.2515
7.252
7.2525
7.253
7.25
Frequency (GHz)
1.0
0.5
0.0
Magnitude
|1⟩
|0⟩
HF
LF
7.2515
7.252
7.2525
7.253
7.25
Frequency (GHz)

Simulation Methodology – Noise

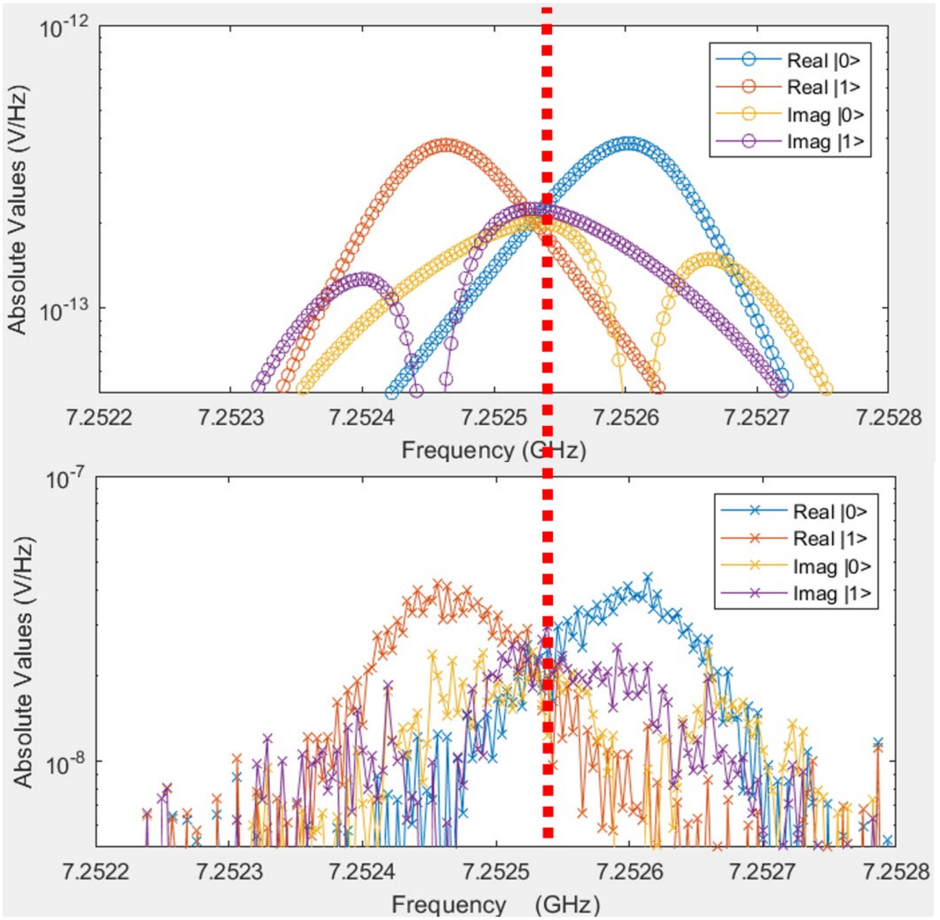




- Thermal Noise of the Amplifiers
  - $4kT_{eff}R$ (HEMT: 1.5K, Last stage: 45K)
  - Amplifier bandwidth used to convert to power
- Photon fluctuation + TWPA noise
  - Modeled as white noise $4kT_nR$
  - $T_n = \frac{1}{\ln 2}\frac{hf}{k}$ (found to be 0.5K)
  - Assume only noise within t_p in the time domain contributes and concentrates in $1/t_p$ in the frequency domain
  - $1/t_p$ used to convert to power

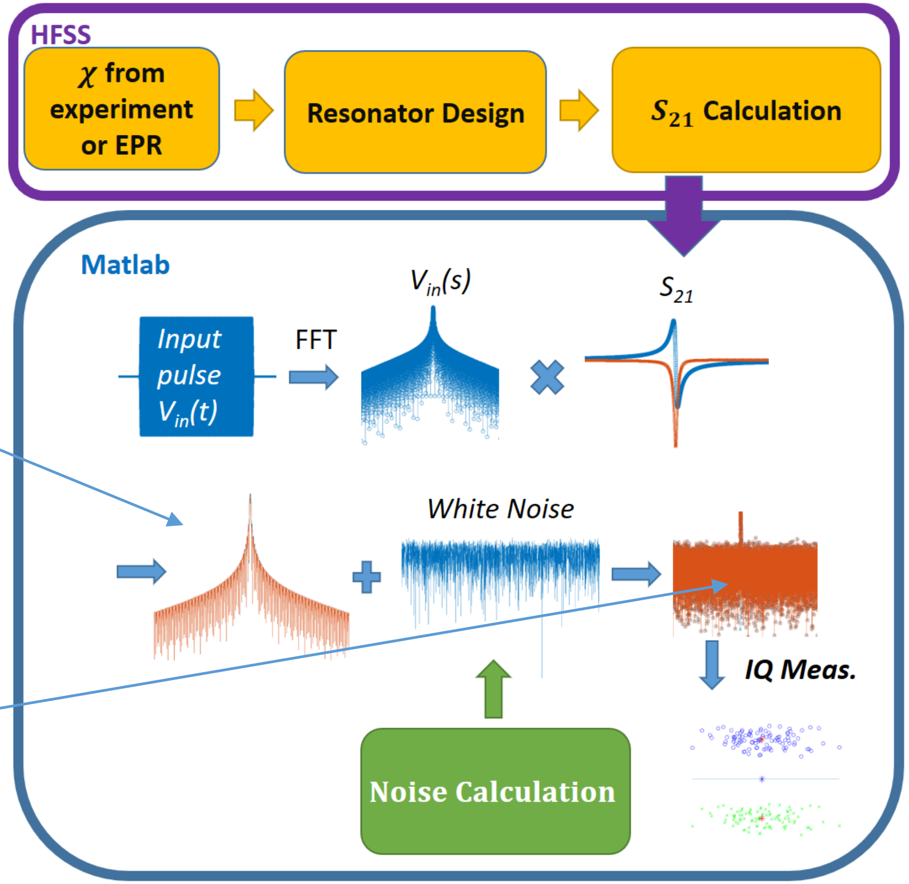
Simulation - Output

$f = 7.252534$ GHz

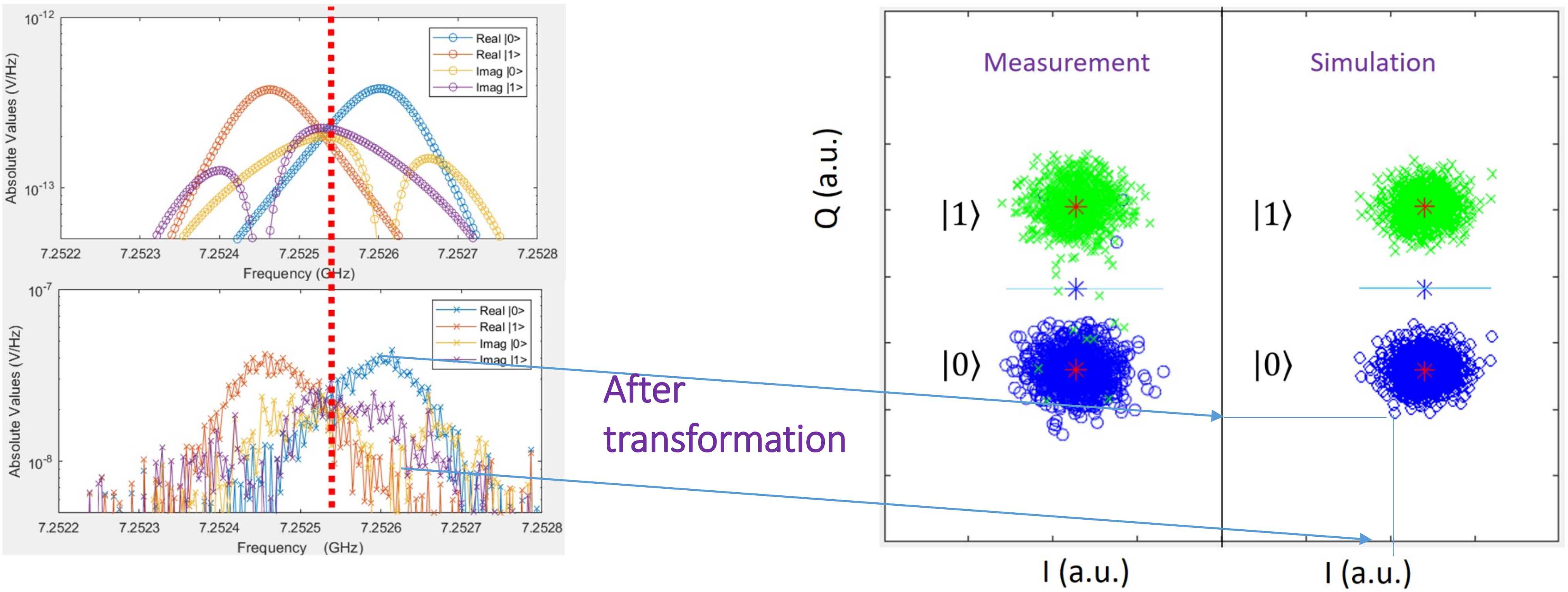


After resonator before adding noise

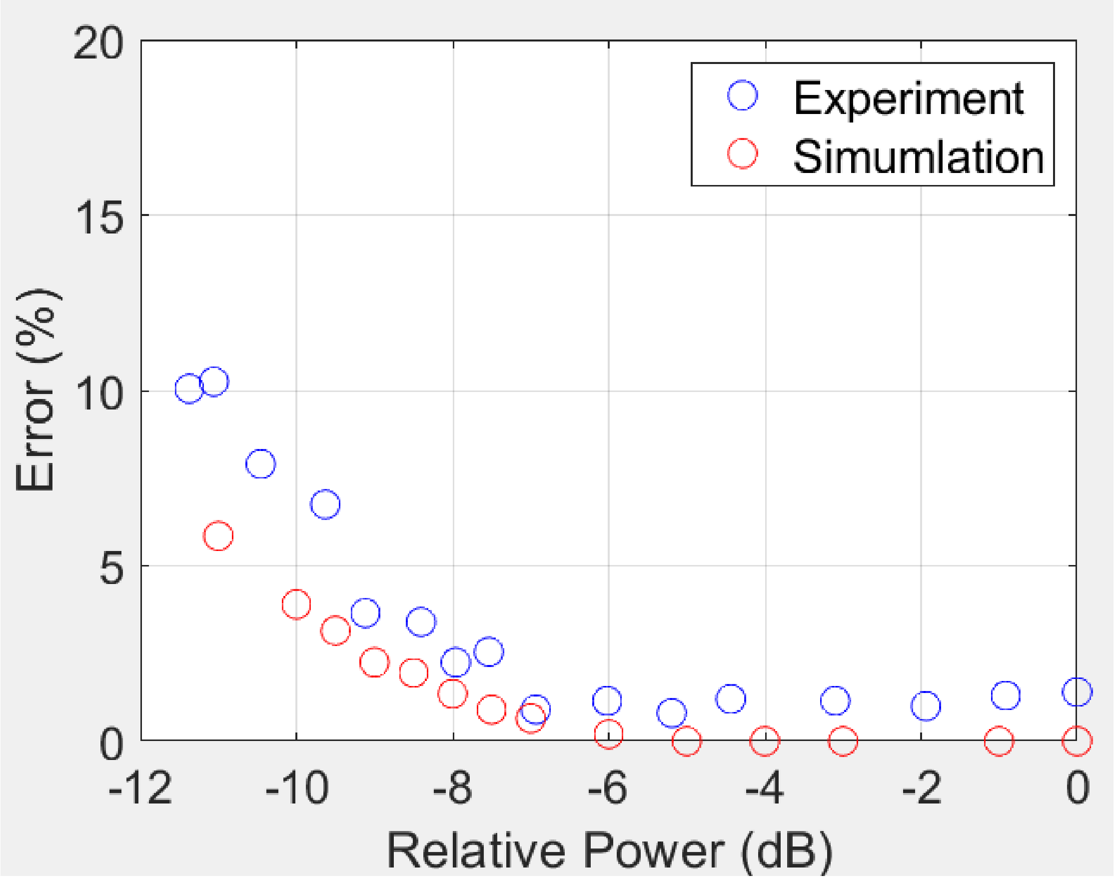
Output



Comparison to Experiment

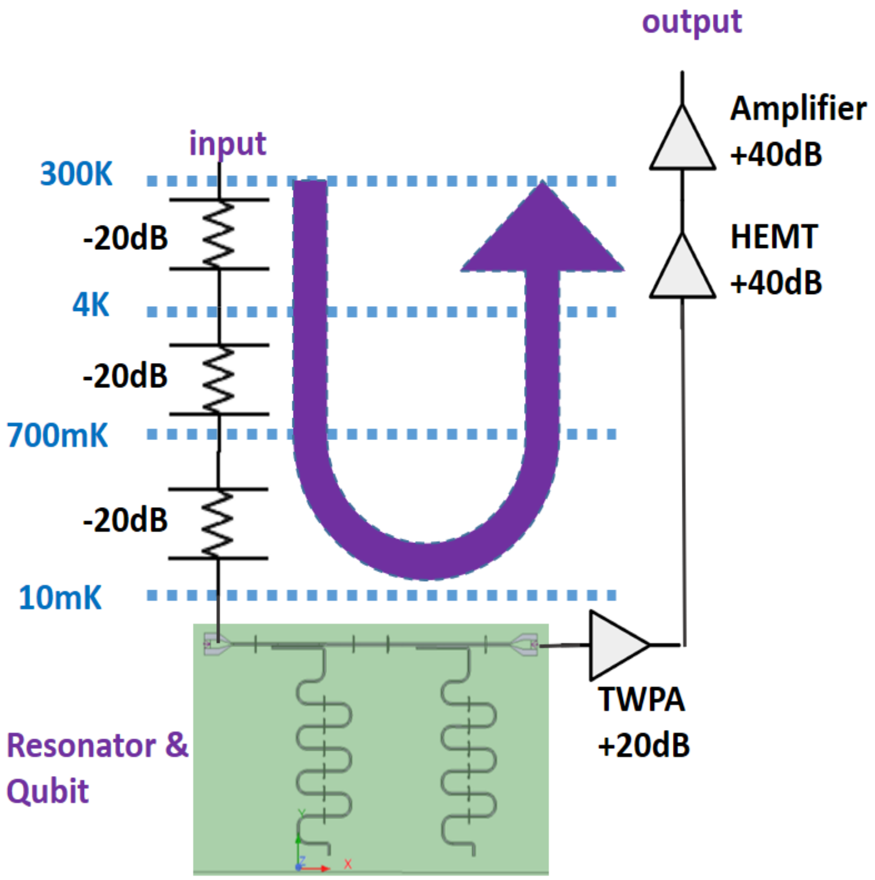


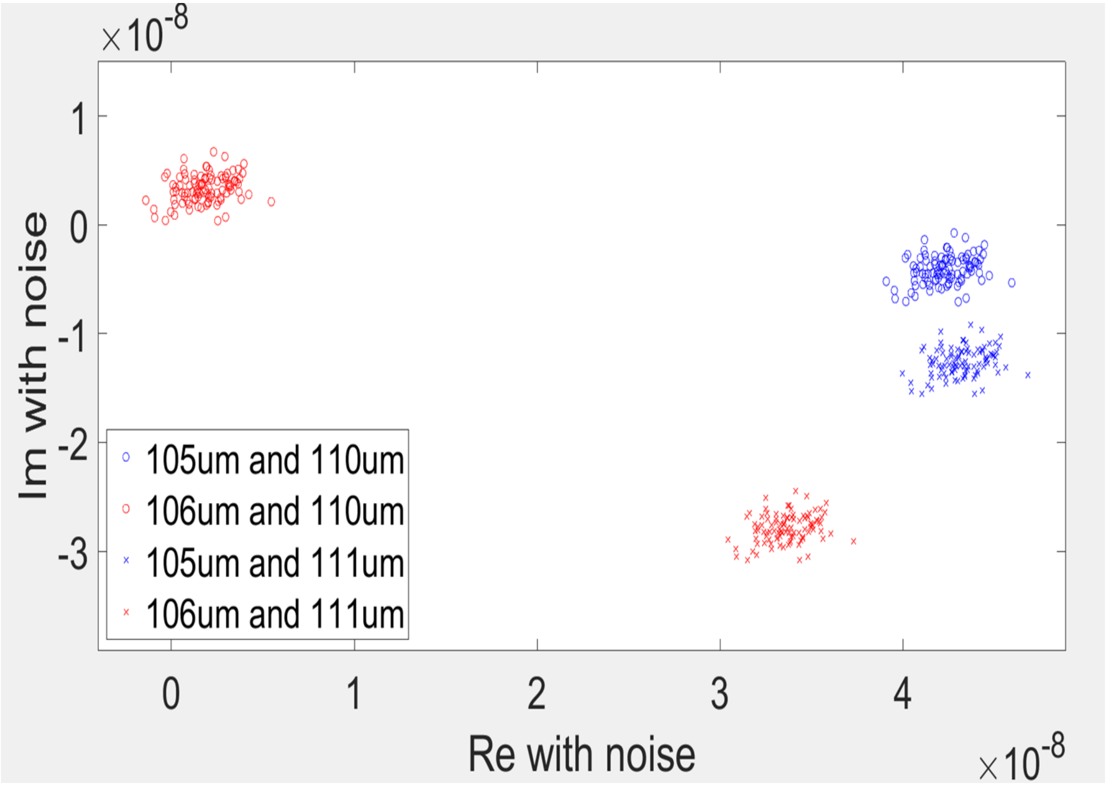
Effect of Readout Pulse Power



- Error increases at relative power = -7dB

Multiple Qubit Readout





Using HHL Algorithm to Solve Linear Equations

- Hector Morrell and Hiu Yung Wong, "Study of using Quantum Computer to Solve Poisson Equation in Gate Insulators," 2021 International Conference on Simulation of Semiconductor Processes and Devices (SISPAD), 2021, pp. 69-72, doi: 10.1109/SISPAD54002.2021.9592604.
- A. Zaman, Hector Morrell, and Hiu Yung Wong, “A Step-by-Step HHL Algorithm Walkthrough to Enhance Understanding of Critical Quantum Computing Concepts,” in IEEE Access, 2023. 10.1109/ACCESS.2023.3297658

Harrow-Hassidim-Lloyd (HHL) Algorithm

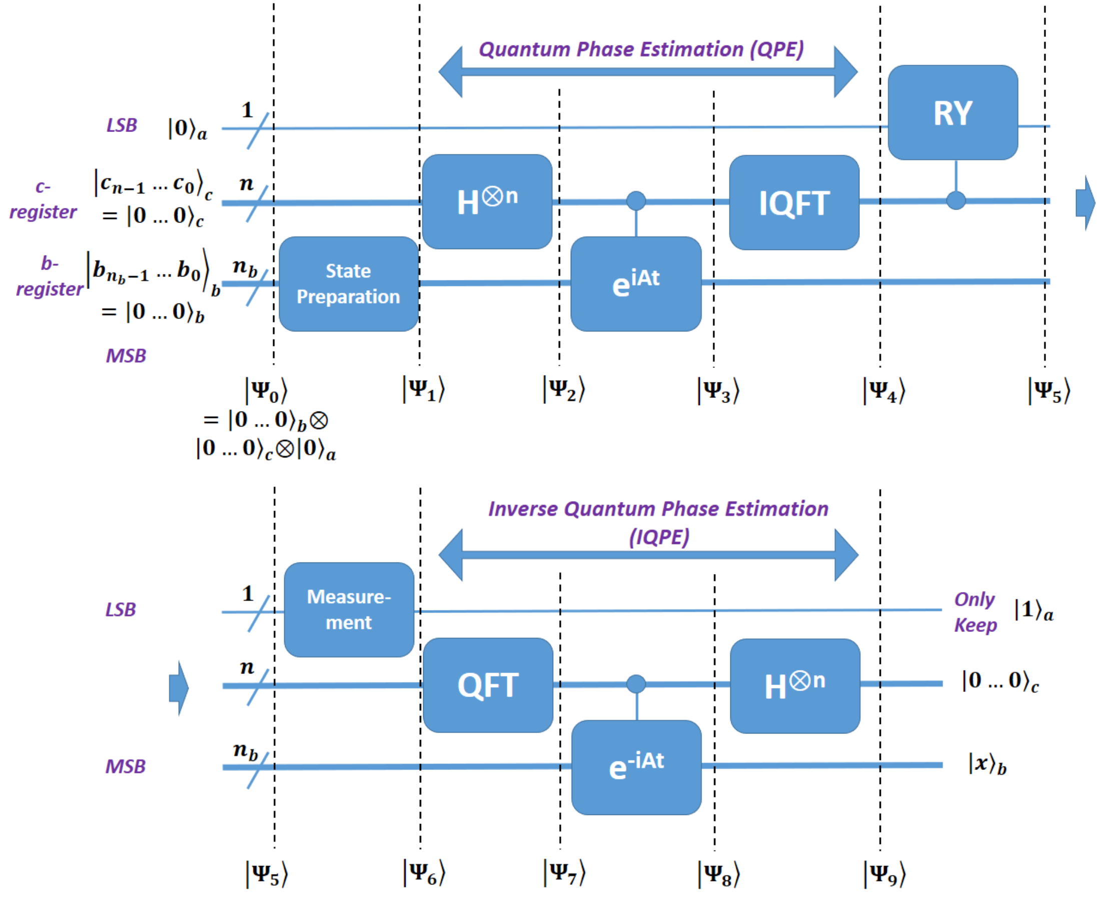
Problem to solve $A\vec{x} = \vec{b}$

$$\vec{x} = A^{-1}\vec{b}$$

$$A = \sum_{i=0}^{2^{n_b}-1} \lambda_i |u_i\rangle \langle u_i|$$

Encoding $|b\rangle = \sum_{j=0}^{2^{n_b}-1} b_j |u_j\rangle$

Solution $|x\rangle = A^{-1}|b\rangle = \sum_{i=0}^{2^{n_b}-1} \lambda_i^{-1} b_i |u_i\rangle$



H. Morell and H. Y. Wong, arXiv:2108.09004v2

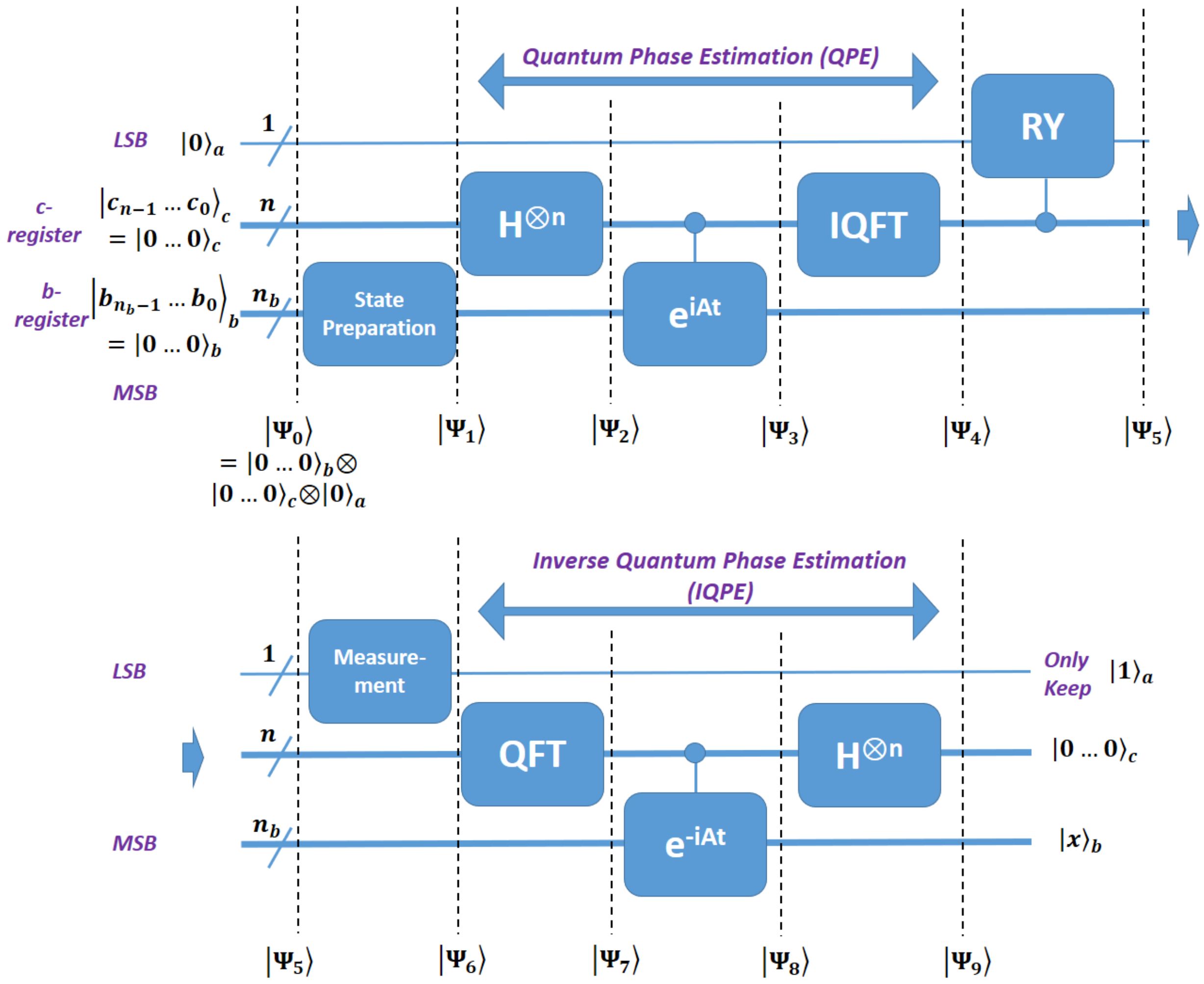
Encoding of $\vec{b}$ and State Preparation

$$\vec{b} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{N_b-1} \end{pmatrix} \Leftrightarrow \beta_0 |0\rangle + \beta_1 |1\rangle + \cdots + \beta_{N_b-1} |N_b - 1\rangle = |b\rangle$$

N dimensional system only needs $log_2 N$ of qubits

State Preparation:

$$|\Psi_1\rangle = |b\rangle_b \, |0 \cdots 0\rangle_c \, |0\rangle_a$$



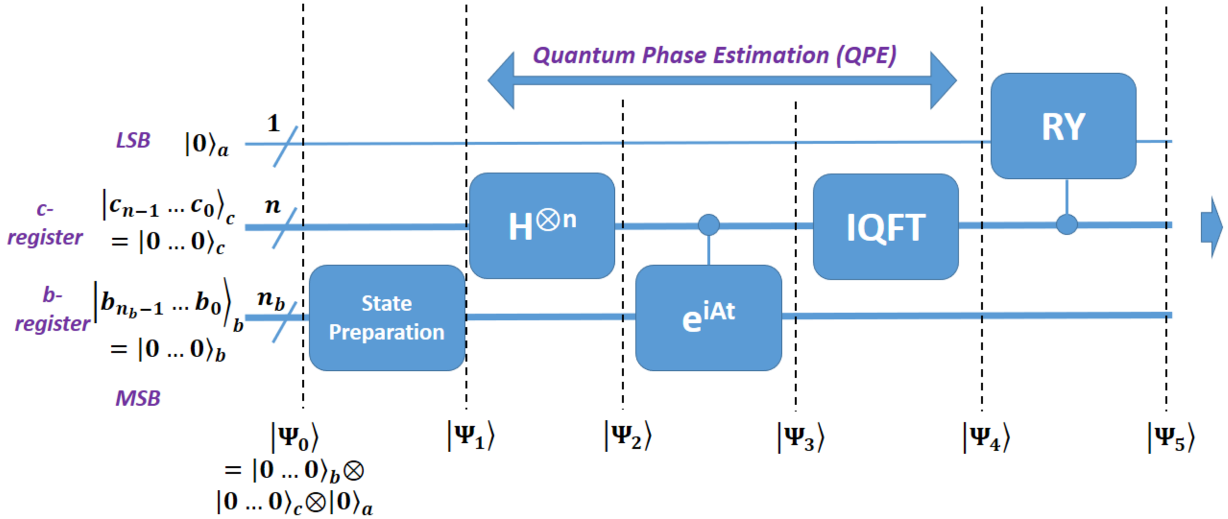
Quantum Phase Estimation

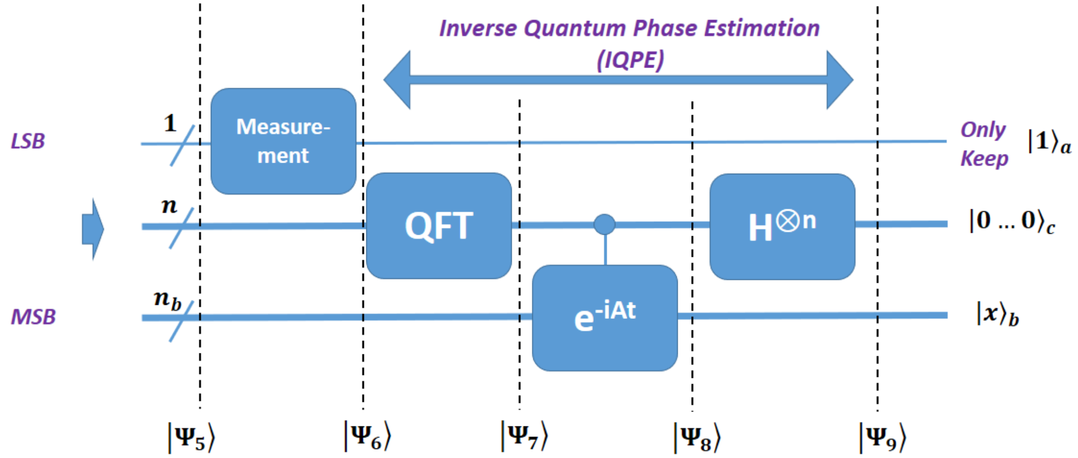
A encoded in the controlled rotation, through Hamiltonian encoding

$$|b\rangle = |u_j\rangle \quad \Rightarrow \quad |\Psi_4\rangle = |u_j\rangle\, |N\lambda_j t/2\pi\rangle\, |0\rangle_a$$

$$|b\rangle = \sum_{j=0}^{2^{n_b}-1} b_j\, |u_j\rangle \quad \Rightarrow \quad |\Psi_4\rangle = \sum_{j=0}^{2^{n_b}-1} b_j\, |u_j\rangle\, |N\lambda_j t/2\pi\rangle\, |0\rangle_a$$

time t and number of qubits in c-register n_l determine the accuracy.

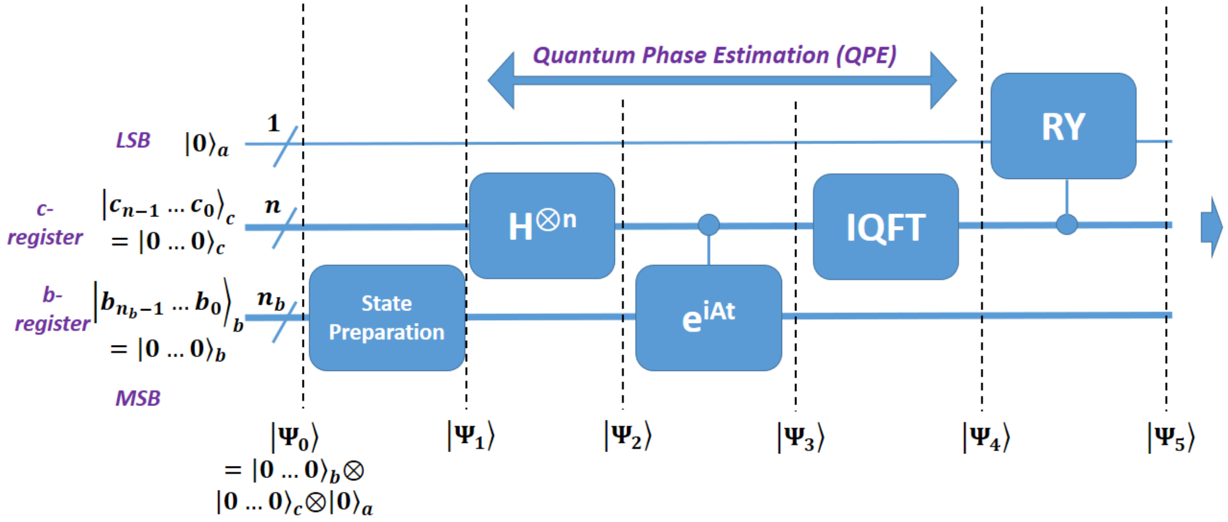




Controlled Rotation

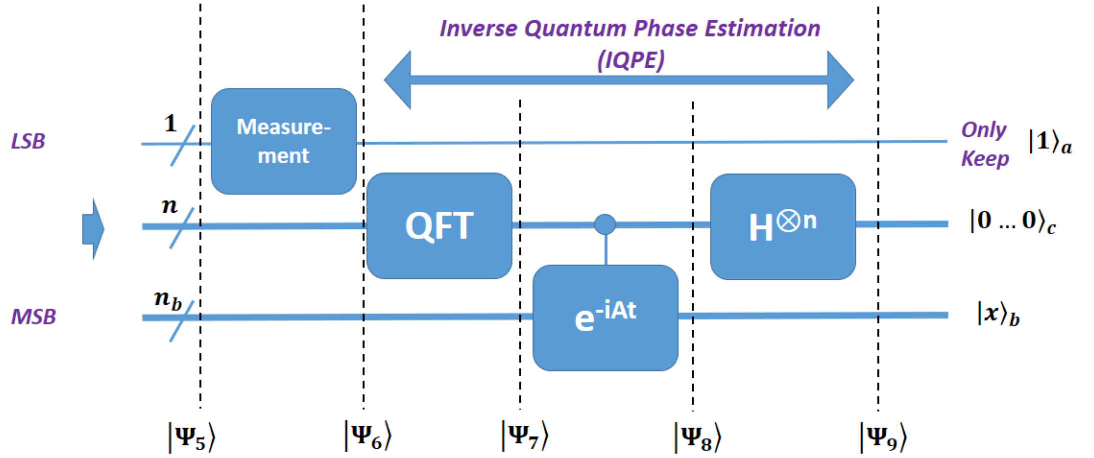
Goal:

$$|\Psi_5\rangle = \sum_{j=0}^{2^{n_b}-1} b_j |u_j\rangle |\tilde{\lambda}_j\rangle \left(\sqrt{1-\frac{C^2}{\tilde{\lambda}_j^{\,2}}}|0\rangle_a + \frac{C}{\tilde{\lambda}_j}|1\rangle_a\right)$$



Measurement (repeat until $|1\rangle_a$):

$$|\Psi_6\rangle = \frac{1}{\sqrt{\sum_{j=0}^{2^{n_b}-1}\left|\frac{b_j C}{\tilde{\lambda}_j}\right|^2}} \sum_{j=0}^{2^{n_b}-1} b_j |u_j\rangle |\tilde{\lambda}_j\rangle \frac{C}{\tilde{\lambda}_j}|1\rangle_a$$



Similar to (but entangled)

$$|x\rangle = A^{-1}|b\rangle = \sum_{i=0}^{2^{n_b}-1} \lambda_i^{-1} b_i |u_i\rangle$$

Disentanglement

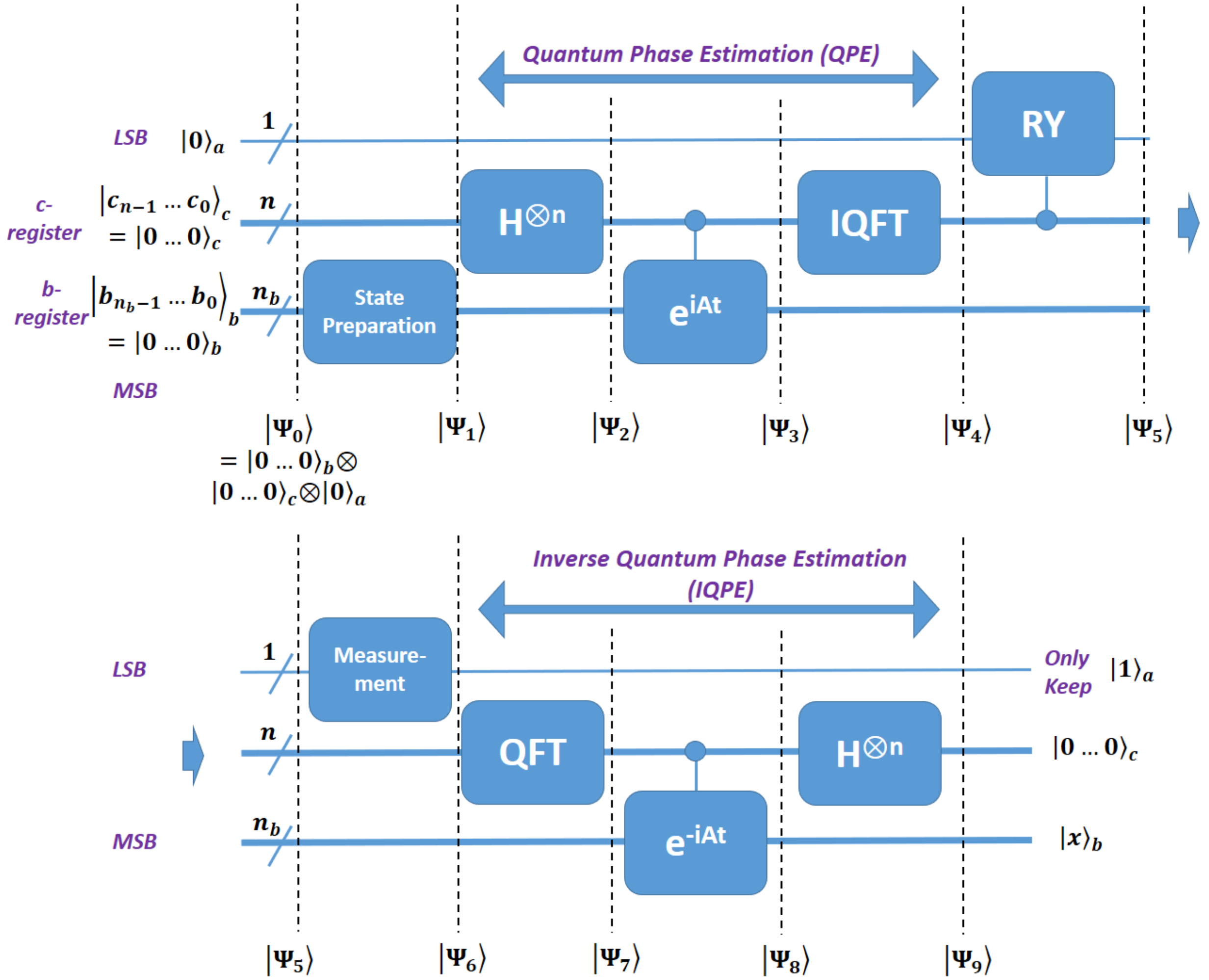
Disentangle using IQPE

$$|\Psi_9\rangle = \frac{1}{\sqrt{\sum_{j=0}^{2^{n_b}-1}\left|\frac{b_j C}{\lambda_j}\right|^2}} \sum_{j=0}^{2^{n_b}-1} \frac{b_j C}{\lambda_j} |u_j\rangle |0\rangle^{\otimes n} |1\rangle_a$$
$$= \frac{1}{\sqrt{\sum_{j=0}^{2^{n_b}-1}\left|\frac{b_j C}{\lambda_j}\right|^2}} |x\rangle_b |0\rangle_c^{\otimes n} |1\rangle_a$$

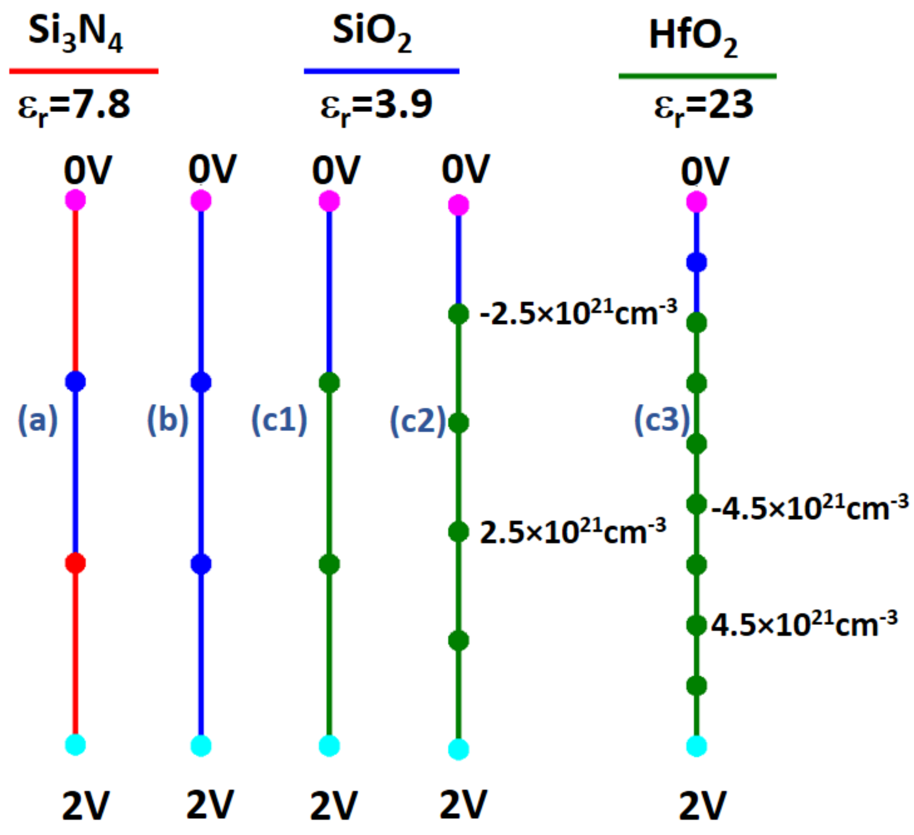
Speed: O(log N)

However, how do we use it as it takes at least O(N) to fetch the result? Only useful if to compute quantities such as

$$\langle\Psi|B|\Psi\rangle$$

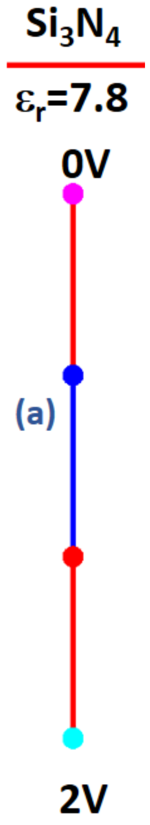


TCAD Problem



Number of qubits in n_b 1 1 1 2 3

Solving Structure a) (circuit 1)



$$\vec{b} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

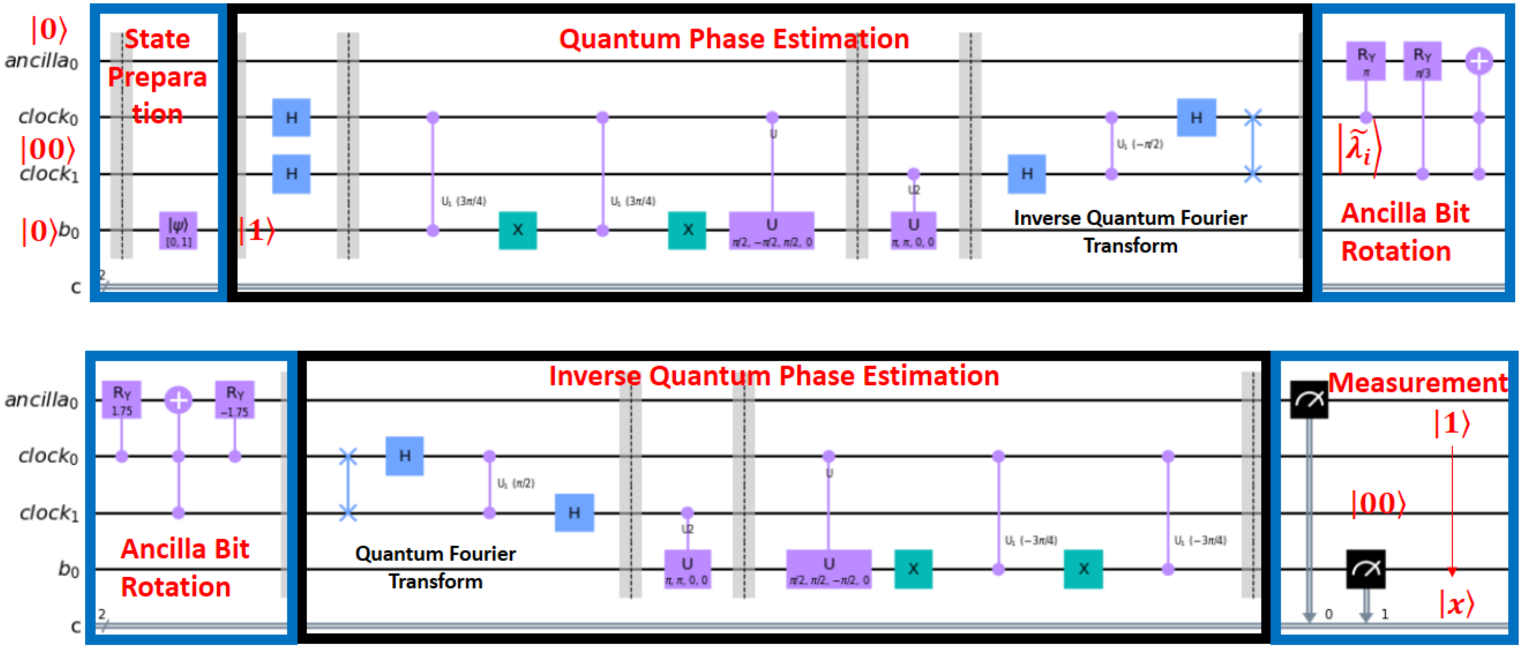
$$\boldsymbol{A} = \begin{pmatrix} 1 & -1/3 \\ -1/3 & 1 \end{pmatrix}$$

$$\lambda_1 = 2/3 \text{ (encoded as } |01\rangle)$$

$$\lambda_2 = 4/3 \text{ (encoded as } |10\rangle)$$

$n_l = 2$ gives exact solution

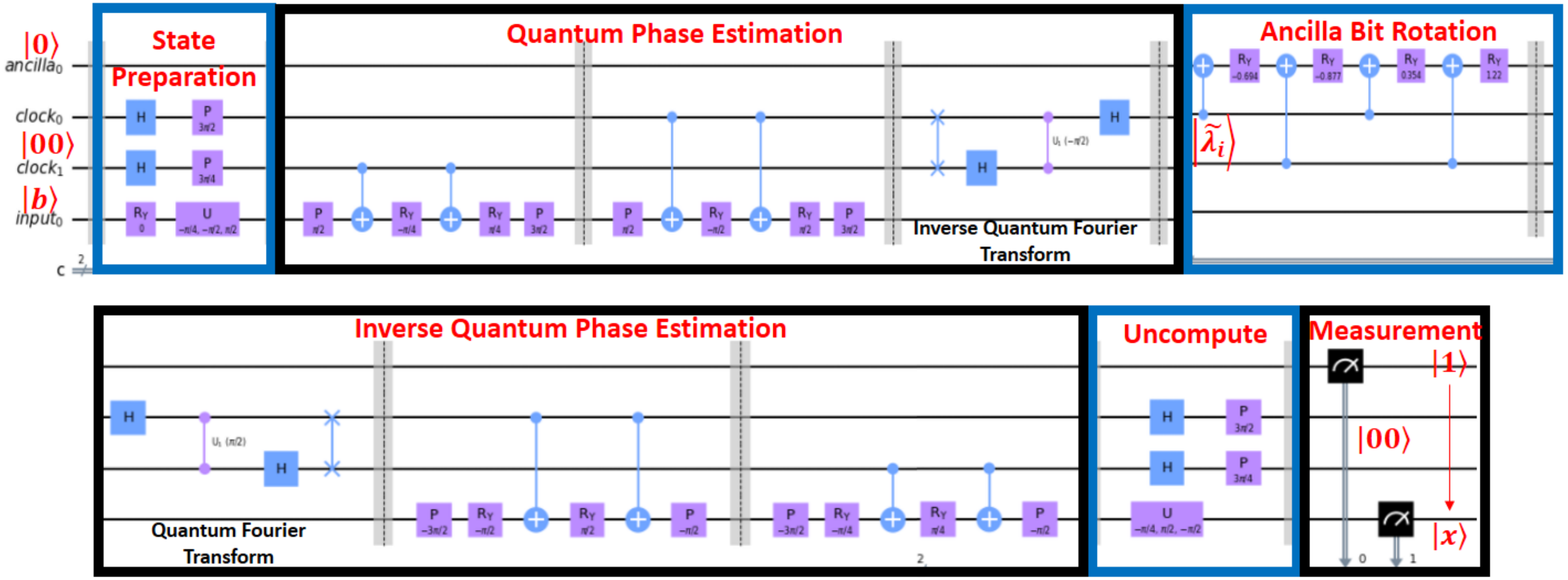
Circuit 1



Based on H. Morell and H. Y. Wong, arXiv:2108.09004v2

Solving Structure a) (circuit 2)

Circuit 2



Based on
https://qiskit.org/textbook/ch-applications/hhl_tutorial.html

Implementation of Controlled Rotation

Controlled rotation $RY(\theta)$ is used to obtain $\frac{C}{\lambda_j}$ in $\sum_{j=0}^{N-1} b_j \left|u_j\right\rangle |00\rangle\left(\sqrt{1-\frac{C^2}{\lambda_j^2}}|0\rangle + \frac{C}{\lambda_j}|1\rangle\right)$

$$RY(\theta) = \exp\left(-i\frac{\theta}{2}Y\right) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -\sin\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$

$$\theta(\widetilde{c_1c_0}) = 2\arcsin\left(\frac{C}{\widetilde{c_1c_0}}\right)$$

Approximated by

$$\theta(\widetilde{c_1c_0}) = \pi c_0 + \frac{\pi}{3}c_1$$

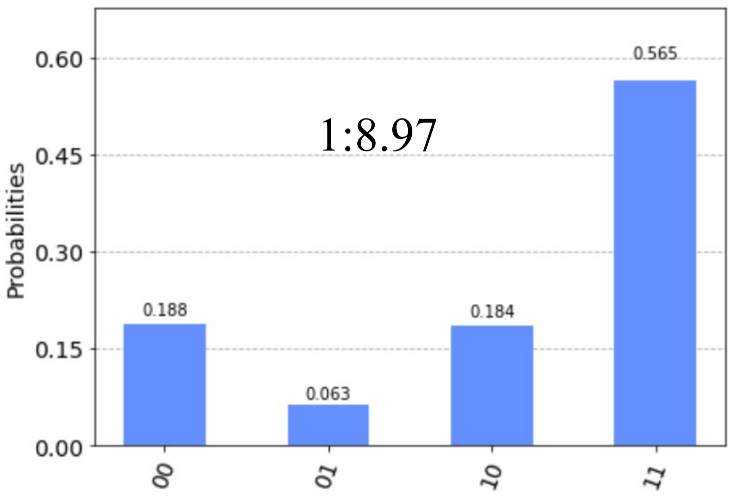
Larger C gives a larger possibility to measure $|\mathbf{1}\rangle_{\boldsymbol{a}}$

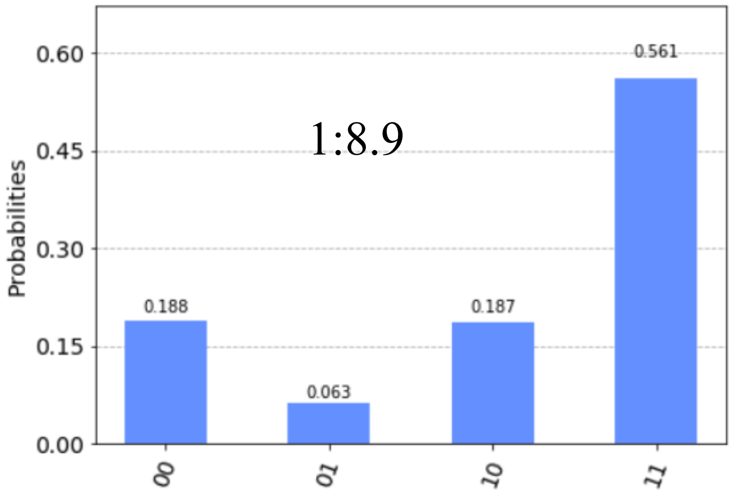
Results

Circuit 1

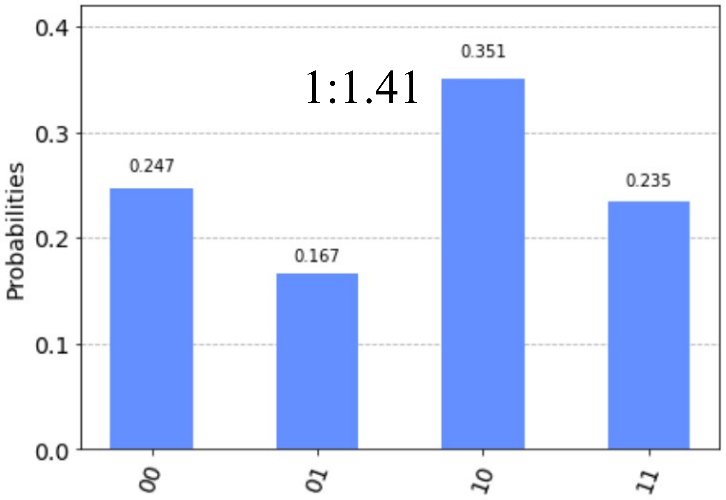
Circuit 2

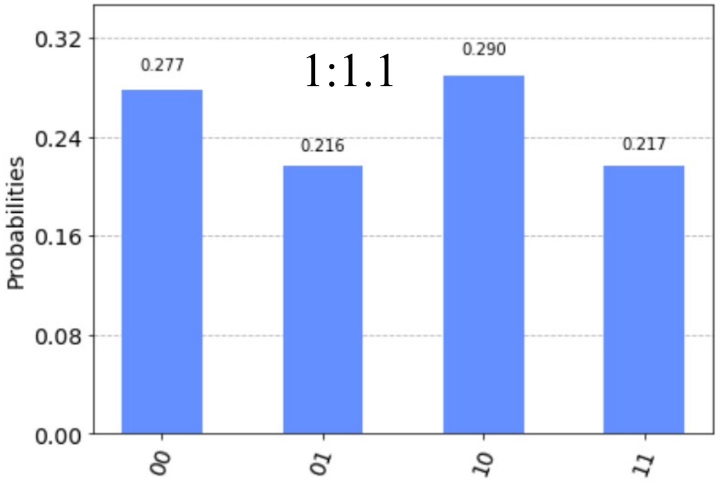
qasm_simulator





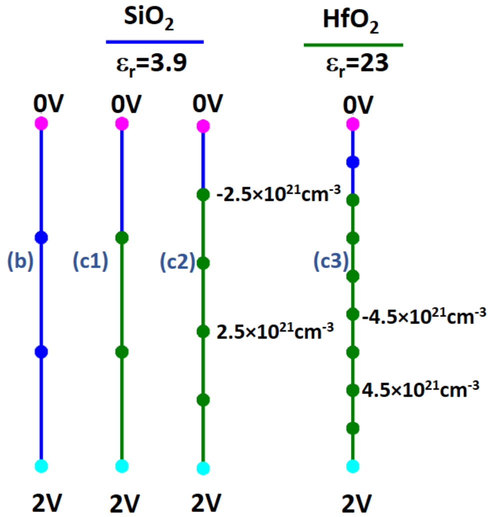
ibmq_5_yorktown

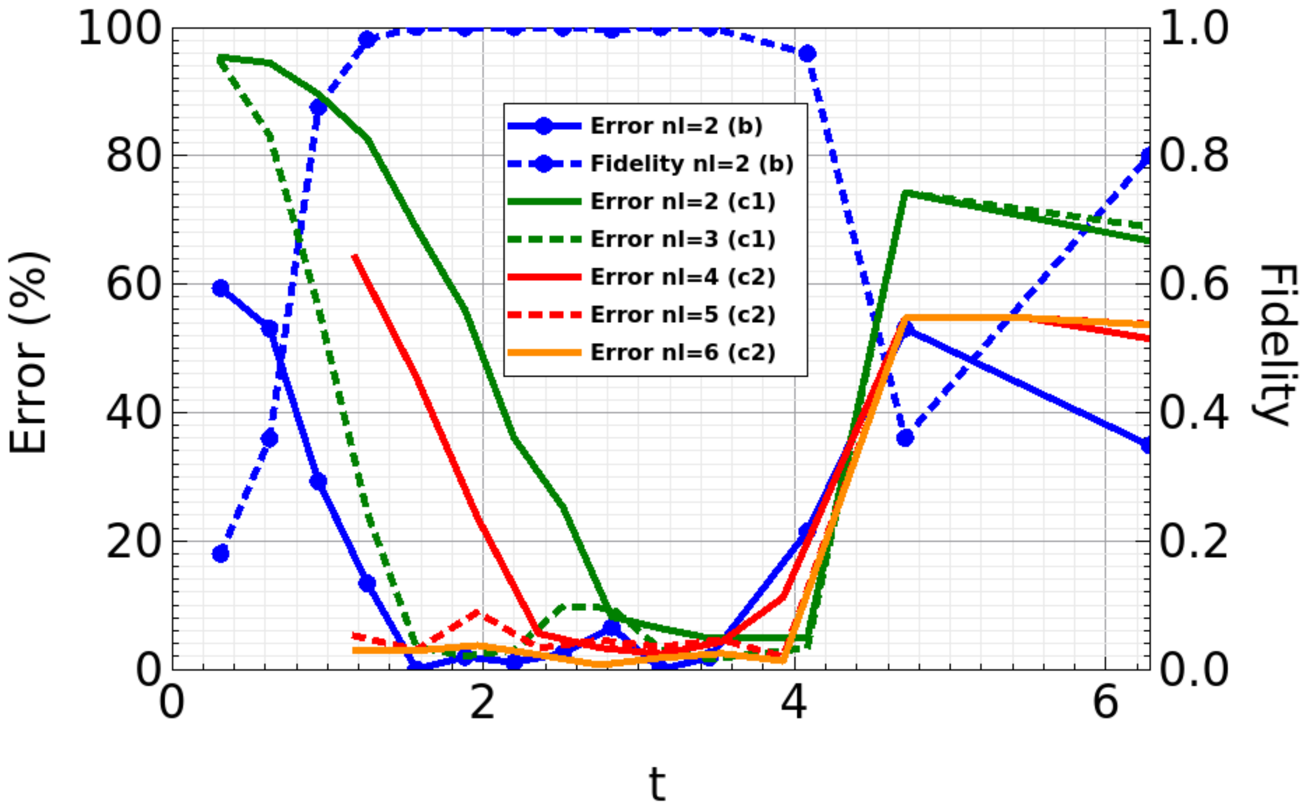


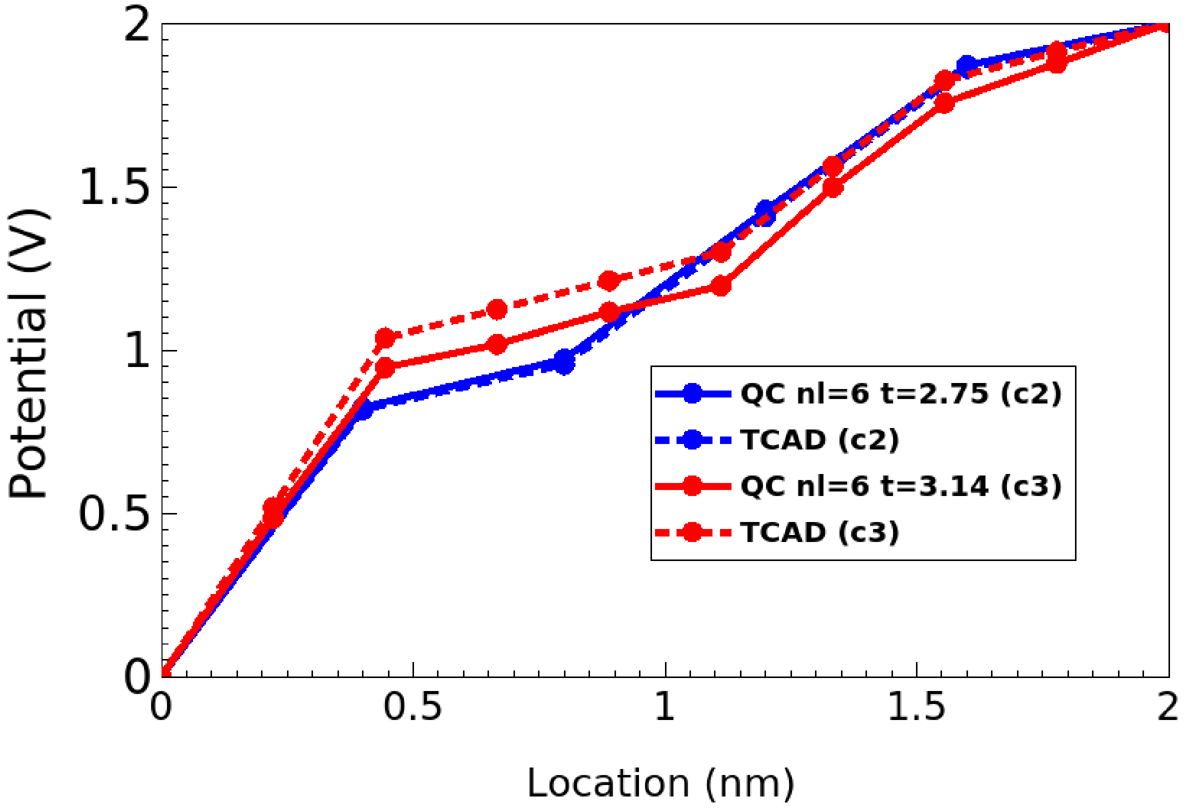


Accuracy of Larger Systems

- Effect of t and n_l
- There is a margin for the time error







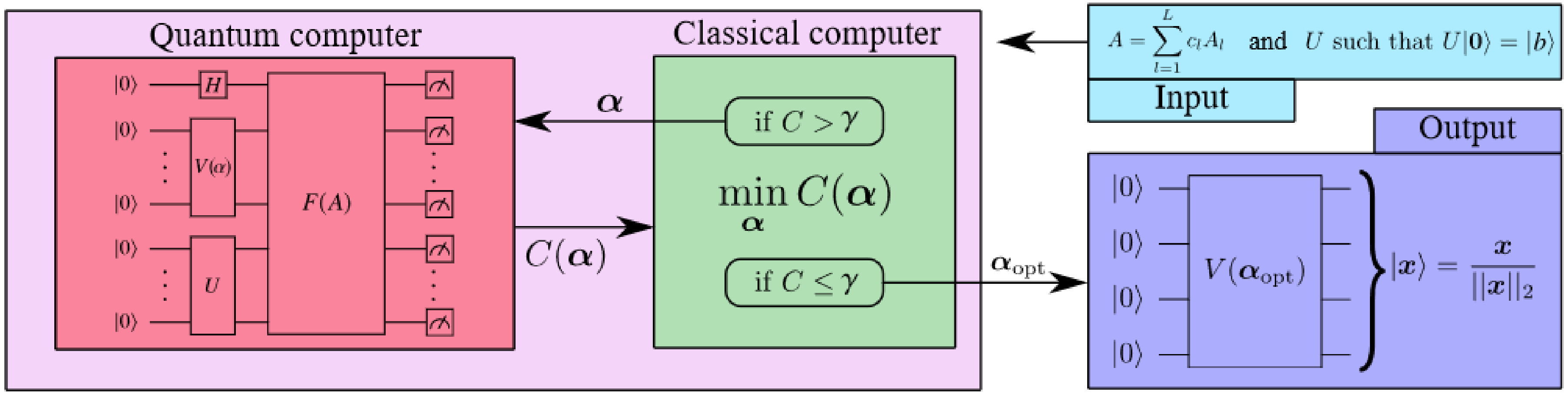
Using Variational Quantum Linear Solver to Solve Systems of Linear Equations

Variational Quantum Linear Solver (VQLS) Algorithm

Problem to solve

$$A\vec{x} = \vec{b}$$

$$\vec{x} = A^{-1}\vec{b}$$



The main concept of VQLS is to use Parameterized or variational hybrid Quantum Classical circuits to solve Linear equation

v(α) — > Variational circuit
α — > Parameters of variational circuit
c(α) — > cost function value

$A = \sum_{l=1}^{L} C_l A_l$ ⟶ Decomposition of A into Linear combination of unitary matrices (A_l) with complex coefficients (C_l)

$\vec{x}$ ⟶ V(α) is temporarily considered as $\vec{x}$ by optimizing α to find $\vec{x}$

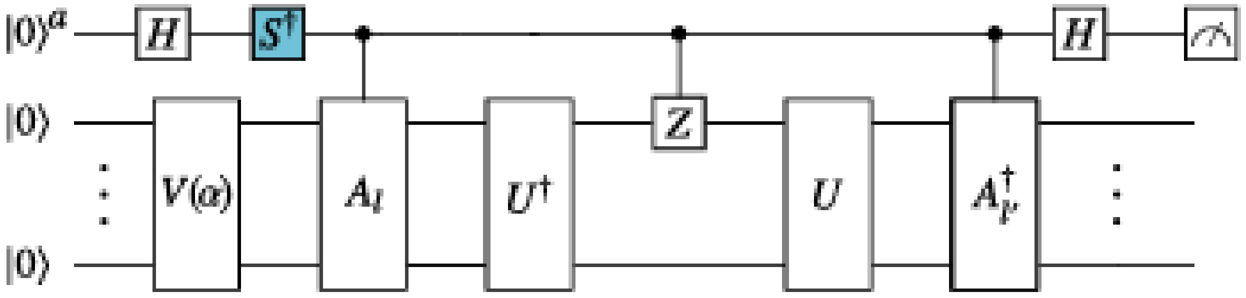
$\vec{b}$ ⟶ Output state defined as unitary matrix (U) applied to |0⟩ state i.e, U|0⟩ = |b⟩

Variational Quantum Linear Solver, arXiv:1909.05820

Variational Quantum Linear Solver (VQLS) Algorithm

$$C_G = \hat{C}_G / \langle\psi|\psi\rangle = 1 - |\langle b|\Psi\rangle|^2$$

$$|\Psi\rangle := \frac{A|x\rangle}{\sqrt{\langle x|A^\dagger A|x\rangle}} \approx |b\rangle.$$



Global cost function $\longrightarrow$ $C_G = 1 - \dfrac{\sum_{l,l'} c_l c_{l'}^* \langle 0|V^\dagger A_{l'}^\dagger U|0\rangle\langle 0|U^\dagger A_l V|0\rangle}{\sum_{l,l'} c_l c_{l'}^* \langle 0|V^\dagger A_{l'}^\dagger A_l V|0\rangle}$

Local cost function $\longrightarrow$ $C_L = \dfrac{1}{2} - \dfrac{1}{2n}\dfrac{\sum_{j=0}^{n-1}\sum_{l,l'} c_l c_{l'}^* \langle 0|V^\dagger A_{l'}^\dagger U Z_j U^\dagger A_l V|0\rangle}{\sum_{l,l'} c_l c_{l'}^* \langle 0|V^\dagger A_{l'}^\dagger A_l V|0\rangle}$

Local or global cost function along with functions like Hadamard test can be used to calculate cost function (normalized expectation value) which is minimized using optimizing parameters using classical computer until converged

Once converged will use optimized parameters to find $\vec{x}$

Simulation Example

1 $A\vec{x} = \vec{b}$

$A =$

```
[[1.  0.  0.  0.  0.4 0.  0.  0. ]
 [0.  1.  0.  0.  0.  0.4 0.  0. ]
 [0.  0.  1.  0.  0.  0.  0.  0. ]
 [0.  0.  0.  1.  0.  0.  0.  0. ]
 [0.4 0.  0.  0.  1.  0.  0.  0. ]
 [0.  0.4 0.  0.  0.  1.  0.  0. ]
 [0.  0.  0.  0.  0.  0.  1.  0. ]
 [0.  0.  0.  0.  0.  0.  0.  1. ]]
```

$$\vec{b} = 0.3535539 \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

2 $A = \sum_{l=1}^{L} C_l A_l$, happened L = 3; Qubit 3 is the ancilla bit as Pennylane is big endian. $C_1 = 1$, $C_2 = 2$, $C_3 = 3$

$A_1 = I$

$A_2 =$

```
3: ─╭●─╭●─
0: ─╰X─│──
1: ────╰Z─
```

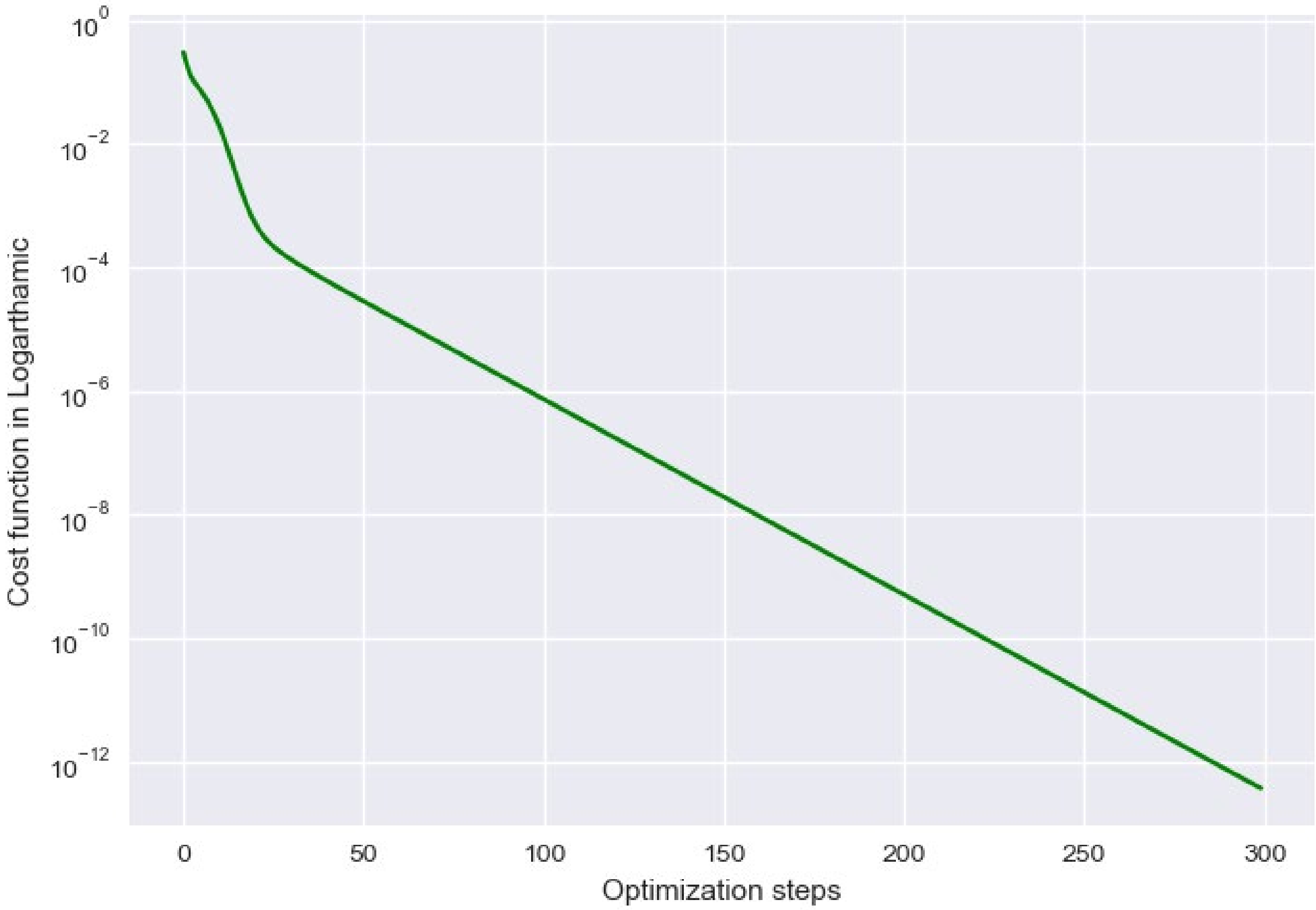
$A_3 =$

```
3: ─╭●─
0: ─╰X─
1: ────
```

3 Ansatz, α are the integers to be varied.

```
0: ──RY(0.00)─╭●──RY(3.00)───────────────╭●──RY(6.00)───────────────╭●──RY(9.00)────────────────┤
1: ──RY(1.00)─╰Z─╭●──────────RY(4.00)─╰Z─╭●──────────RY(7.00)─╰Z─╭●──────────RY(10.00)─┤
2: ──RY(2.00)────╰Z──────────RY(5.00)────╰Z──────────RY(8.00)────╰Z──────────RY(11.00)─┤
```

Solution



VQLS:

$|<x|n>|2$
$= [0.0877\ 0.088\ \ 0.1639\ 0.1673\ 0.0847\ 0.081\ \ 0.163\ \ 0.1644]$

Classical:
$|<x|n>|2$
$= [0.0845\ 0.084\ 0.1655\ 0.1655\ 0.0844\ 0.084\ 0.166\ 0.1655]$

What happened in the last 20 minutes?

- Self-Introduction
- Overview of Quantum Computing
  - Basics
  - Hardware
- Simulation for Quantum Computers
- Quantum Computing for Simulation
  - HHL Algorithm
  - Variational Quantum Linear Solver

Acknowledgement

- Some of the materials are based upon work supported by the National Science Foundation under Grant No. 2046220 and Grant No. 2125906
- The research work is benefited from the “SJSU-IBM Acceleration: Quantum Classrooms” project.

Quantum Classrooms

SJSU-IBM Acceleration: Quantum Classrooms